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ABSTRACT

This course in energy economics is one of 16 courses in the Energy Technology Series developed for an Energy Conservation-and-Use Technology curriculum. Intended for use in two-year postsecondary technical institutions to prepare technicians for employment, the courses are also useful in industry for updating employees in company-sponsored training programs. Comprised of five modules, the course is designed to familiarize the student with the energy-conserving and cost-saving measures that are available, as well as the analysis techniques that are necessary for accurate evaluation of energy projects. Written by a technical expert and approved by industry representatives, each module contains the following elements: introduction, prerequisites, objectives, subject matter, exercises, laboratory materials, laboratory procedures (experiment section for hands-on portion), data tables (included in most basic courses to help students learn to collect or organize data), references, and glossary. Module titles are Fundamentals of Energy Cost Analysis, Financial Parameters of Energy Economics, Financial Techniques of Energy Economics, Economics of Energy Alternatives, and Economic Analysis and Energy Conservation Projects. (YLB)

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ENERGY
ECONOMICS

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PREFACE

ABOUT ENERGY TECHNOLOGY MODULES

The modules were developed by TERC-SW for use in two-year postsecondary technical institutions to prepare technicians for employment and are useful in industry for up-dating employees in company-sponsored training programs. The principles, techniques and skills taught in the modules, based on tasks that energy technicians perform, were obtained from a nationwide advisory committee of employers of energy technicians. Each module was written by a technical expert and approved by representatives from industry.

A module contains the following elements:

Introduction, which identifies the topic and often includes a rationale for studying the material.

Prerequisites, which identify the material a student should be familiar with before studying the module.

Objectives, which clearly identify what the student is expected to know for satisfactory module completion. The objectives, stated in terms of action-oriented behaviors, include such action words as operate, measure, calculate, identify and define, rather than words with many interpretations, such as know, understand, learn and appreciate.

Subject Matter, which presents the background theory and techniques supportive to the objectives of the module. Subject matter is written with the technical student in mind.

Exercises, which provide practical problems to which the student can apply this new knowledge.

Data Tables, which are included in most modules for the first year (or basic) courses to help the student learn how to collect and organize data.

References, which are included as suggestions for supplementary reading/viewing for the student.

Glossary, which defines and explains terms or words used within the module that are uncommon, technical, or anticipated as being unfamiliar to the student.

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ENERGY ECONOMICS

INTRODUCTION

In recent years the prices of energy have increased faster than the prices of most other commodities. Just how much they have increased can be seen by examining the following table.

U.S. NATIONAL AVERAGE PRICES
OF DIFFERENT SOURCES OF ENERGY IN RECENT YEARS.

Year	Residential Heating Oil (¢ per gal)	Gasoline (¢ per gal)	Residential Electricity (¢ per kWh)
1973	28.4	38.8	2.38
1976	41.1	58.6	3.45
1977	45.2	62.6	4.03
1978	48.0	64.6	4.10
1979	64.2	81.2	4.21

Energy users are beginning to realize that some sources of energy, such as oil and natural gas, are going to be more difficult and more expensive to find and produce. As a result, there is an increased interest in finding ways to reduce energy use. Whenever energy use is reduced, it is said that energy has been conserved.

Too much of a reduction of energy use, however, can produce adverse effects. For example, it might be acceptable to raise the temperature in an office building by 5°F during the cooling season in order to reduce energy use. The result, of course, would be that the energy-using cooling unit would turn on less frequently. On the other hand, if the temperature were to be raised even further, thereby conserving even more energy, the people who work in the building would become uncomfortable. In this case, work effectiveness would suffer and the energy conservation measure would not be wise. Thus, every energy conservation measure should be analyzed to determine exactly how much energy it will save and what its total effects will be.

The energy specialist can increase the financial well-being of an individual or an organization by analyzing and implementing methods for decreasing the amount of money that is spent for energy. Many opportunities to reduce energy costs require that money be spent for energy-saving devices. For example, typical devices include insulation or a microcomputer that will control room temperatures. However, the cost of the project must be compared to the energy savings it might generate. This, and other such similar tasks, is the job of an energy specialist.

Energy Economics is a course designed to familiarize the student with the energy-conserving and cost-saving measures that are avaliable, as well as the analysis techniques that are necessary for accurate evaluation of energy projects. The course contains the following modules:

Module EE-01, "Fundamentals of Energy Cost Analysis," introduces the student to some of the fundamental principles of economics and shows how these principles can be applied to energy conservation and use. Because analyses must be made of the long-range economics benefits realized in energy conservation projects with regard to their costs and the cost savings they generate, different kinds of costs are defined, and an explanation is given of how price is determined.

Module EE-02, "Financial Parameters of Energy Economics," introduces the student to several fundamental techniques used to analyze the costs and cost savings of energy projects. The important relationship that exists between time and money is explained in this module.

Module EE-03, "Financial Techniques of Energy Economics," discusses analysis techniques that apply primarily to recurring costs and cost savings, with particular emphasis on when and how often these costs and cost savings occur. The distinction between recurring and one-time costs was made in previous modules. With the skills that can be learned from the study of this module, the energy specialist will be able to compute the present value of costs and cost savings, an important consideration in energy projects.

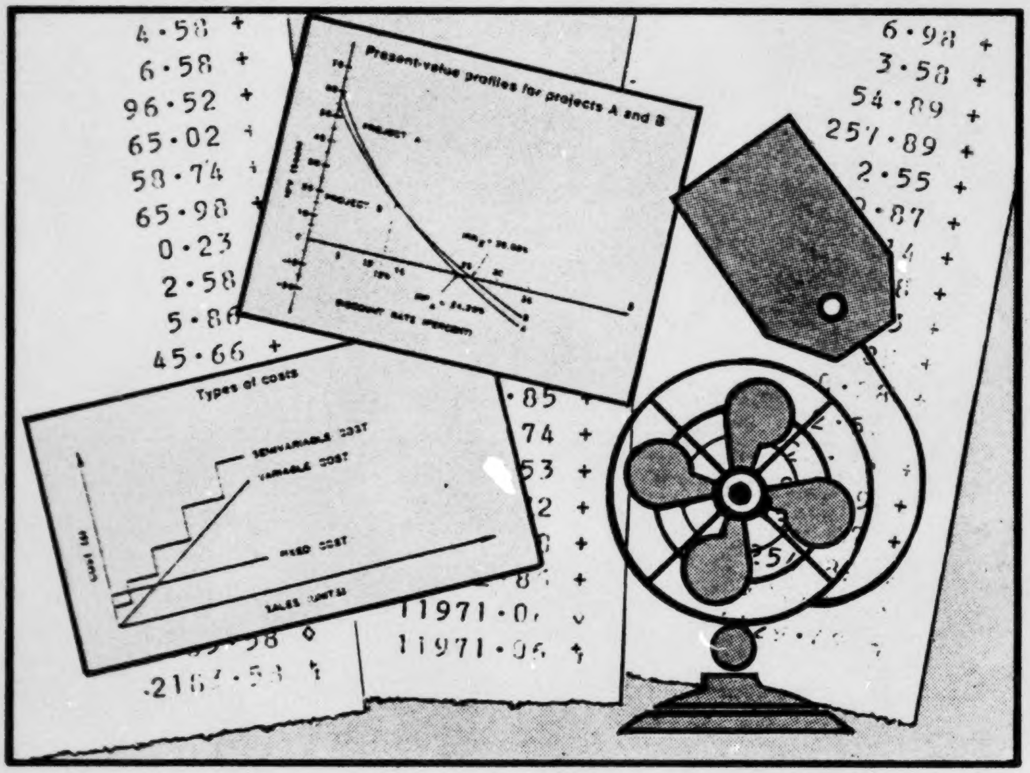
Module EE-04, "Economics of Energy Alternatives," introduces the student to several factors that can affect the actual level of the costs and cost savings of energy projects. Often the actual level is not what it initially appears to be. The process of borrowing money to finance an energy project is discussed, and the impact on costs is demonstrated. Taxes and their effect on costs and cost savings are explored in this module, as well as the concept of life-cycle costing.

Module EE-05, "Economic Analysis and Energy Conservation Projects," presents several techniques that are used in the analysis of ways in which costs and cost savings of energy conservation projects are related. The calculations involved in these techniques are explained and demonstrated. The information needed to perform each calculation and the results of each calculation are emphasized, as well as when each method should be used. With these techniques, the energy specialist should be able to analyze accurately the economic effects of most energy conservation projects or groups of projects.

ENERGY TECHNOLOGY

CONSERVATION AND USE

ENERGY ECONOMICS



EE-01

FUNDAMENTALS OF ENERGY COST ANALYSIS

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INTRODUCTION

There are two major reasons for reducing energy use: one is to conserve available energy resources, such as oil, natural gas, and so forth; and the other is to save money. Energy savings can be accomplished in at least three ways:

1. By changing organizational procedures, such as working hours, or personal habits, such as lowering the setting on a thermostat for controlling building heat in the winter.
2. By modifying an existing building, such as adding storm windows, or changing techniques on new buildings, such as increasing wall or ceiling insulation.
3. By replacing or modifying equipment used for production and/or building climate control.

Although these methods may result in an immediate savings of energy consumption, they frequently require increased costs to alter design techniques and/or organizational procedures. Because of these facts, an analysis must be made of the long-range economic benefits realized in energy conservation projects with regard to their costs and the cost savings they generate.

This module introduces the student to some of the fundamental principles of economics and shows how these principles can be applied to energy conservation and use. Different kinds of costs are defined, and an explanation is given of how price is determined.

PREREQUISITES

The student should have a good understanding of basic algebraic functions.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Define the following terms:
    a. Profit.
    b. Total revenue.
    c. Total cost.
    d. Profit maximization.
    e. Cost minimization.
    f. Energy economics.
    g. Indirect cost.
    h. Direct cost.
    i. One-time cost.
    j. Recurring cost.
    k. Efficiency.
    l. Economic efficiency.
    m. Free market.
    n. Supply.
    o. Resource.
    p. Demand.
    q. Market price.
    r. Marginal cost.
    s. Marginal cost savings.
2. Distinguish between the following kinds of costs:
    a. Direct and indirect.
    b. One-time and recurring.
3. Given the proper information, determine the following:
    a. Market supply.
    b. Market demand.
    c. Market price.
4. Determine the effect of any change in total revenue and/or total cost on profit.

5. Examine energy conservation projects and energy-using systems for economic efficiency.
6. Determine the effect of changes in supply and/or demand on market price.
7. Use marginal analysis (when appropriate) to determine how much should be spent on an energy conservation project.

BUSINESS FIRMS AND ENERGY CONSERVATION

The primary goal of most businesses is to make money. The money-making process usually involves a number of factors that must be considered. Many of these considerations also can apply to individuals or to organizations having other goals (such as non-profit organizations – which will be discussed later). By applying conservation techniques in those areas where energy is used improperly, the energy specialist can help a business achieve its primary goal.

PROFIT, REVENUE, AND COST

The amount of money a business makes is its profit. Profit is defined as "the amount of money people pay a business for the goods or services it sells (total revenue), minus the amount of money the business must pay out in operating expenses (total cost)." In other words, when a business sells something, the money it receives is revenue; when it buys something, such as the electricty for the building or the labor of an employee, the money it spends is a cost. The purpose of operating a business is for the business to receive more in revenue than it spends in costs; this difference is called profit. The mathematical definition of profit is given by Equation 1:

$$\pi = TR - TC \qquad \text{Equation 1}$$

where:

π = Profit.
TR = Total revenues (the amount a business receives from selling goods or services).
TC = Total costs (the amount a business pays to continue its operations).

This mathematical definition points out several important aspects of profit: (1) profits can have a positive, negative, or zero value; (2) profits can be increased by increasing total revenue, decreasing total cost, increasing revenue more than increasing cost, or decreasing cost more than decreasing revenue; and (3) profit maximization occurs when the difference between total revenue and total cost is the greatest. The relationship among these three amounts is given in Table 1.

Most business firms in the United States today have a level of profit that is greater than zero. A firm with negative profit is losing money – it is spending more than it is taking in. In this situation, a firm must do one of two things: (1) change revenues and/or costs so profit is no longer less than zero, or (2) go out of business. Many firms can – and do – lose money for a brief period of time, but they must show a profit eventually or go out of business.

How does the energy specialist help a business reduce costs? In most cases the specialist will have very little to say about factors that have an impact on revenues. What a firm makes to sell, the quantity it makes, and the price it charges are decisions made by management personnel and, therefore, should be of little concern to the energy specialist. Cost considerations are another matter. By helping the firm minimize the amount of case it must spend to stay

TABLE 1. CORRELATION BETWEEN PROFITS, REVENUES, AND COSTS.

↑ TR ↔ TC	↑ π
↓ TC ↔ TR	↑ π
↑ TR > ↑ TC	↑ π
↓ TC > ↓ TR	↑ π
↓ TR ↔ TC	↓ π
↑ TC ↔ TR	↓ π
↓ TR > ↓ TC	↓ π
↑ TC > ↑ TR	↓ π
↑ TR = ↑ TC	↔ π
↑ TC = ↓ TR	↔ π

TR = Total Revenue
TC = Total Cost
π = Profit
↑ = Increase
↓ = Decrease
↔ = No Change
> = Greater than

in business, the energy specialist can help the firm make a larger profit. This is called cost minimization. The energy specialist can help a firm minimize costs in the following ways:

- By determining which particular method of accomplishing a certain task (such as illuminating a building) will use the least energy and, therefore, cost the least amount of money.

- By spotting and evaluating ways to conserve energy in systems already being utilized.

MEASUREMENT OF ENERGY CONSERVATION

All efforts to reduce energy use must be measurable by some common unit so various energy-saving alternatives can be compared. That common unit is dollars. Whereas some other units, such as Btus, could be used just as easily, the conversion of all energy savings to dollar figures makes their value to the business firm much easier to interpret. Thus energy economics can be defined as "the dollar comparison between various energy conservation alternatives to determine the extent of energy savings (reduced costs) that each alternative will produce in relation to the dollars expended to put the alternative in place."

INDIVIDUALS AND NON-PROFIT ORGANIZATIONS

The principle of cost minimization also can be applied to individuals and to non-profit organizations such as churches, hospitals, and schools. In the case of an individual, the less money spent on energy, such as for electricity or for gasoline, the more money there is to spend on something else. A non-profit organization usually exists to perform some type of service, such as in medical care or in education. The less money such an organization must spend on energy, the more money it can spend to provide its services. In either case, keeping energy costs as low as possible is desirable; therfore, the energy specialist's goal is always the same: to reduce energy use to minimize costs.

TYPES OF COSTS

Energy, in most cases, does not have the same demand-price relationship as other products. With government regulation and price control, cost is not totally due to market-demand. Increases in electricity cost is not reflective of increased demand, but due to fuel conversion, pollution control, increases in fuel costs, and other outside factors.

Any device, equipment, or system that uses energy will have an initial purchase cost and, in most cases, an operation cost. A heating, ventilating, and air conditioning (HVAC) system will cost a certain amount to purchase and install; then energy and maintenance costs will have to be paid to operate the system. Quite often, an energy-conservation project will also have an initial implementation cost.

Cost can be classified as direct costs or indirect costs. Direct costs are those that involve the actual spending of money. For instance, when money is spent to buy solar panels for a house, the money spent is a direct cost. An indirect cost is incurred when a business or individual performs a task rather than paying to have it performed. Suppose an individual installs a solar water-heating system rather than hiring an HVAC contractor to do the job. Even though the individual does not pay to install the system, there is still a cost involved since the individual could be earning money during the time used to install the system. The amount of money that could have been earned is the indirect cost of installing the unit. Indirect costs are much more difficult to compute than direct costs; nevertheless, they are important and should be considered.

Direct costs can be categorized as follows: one-time costs and recurring costs. A one-time cost is exactly what the term implies – something that is paid just once. The purchase price of a new light fixture is an example of a one-time cost. Recurring costs are costs that are paid on a regular basis over a specified period of time. The cost for electricity required to operate an HVAC system is a recurring cost, since it must be paid each month. The costs associated with an energy system can be different for different systems, so the energy specialist should consider each situation carefully and classify each cost properly.

EXAMPLE A: TYPES OF COSTS.

Given: The following costs are incurred by the owner of a building for a heating and cooling system:

a. Filters
b. 5-ton cooling system
c Duct work
d. Belts
e. Gas furnaces and evaporative coils
f. Installation – slab wiring
g. Salaries of maintenance personnel
h. Cost of power: electricity and natural gas

Find: Whether the preceding costs are one-time or recurring.

Solution: One-time costs:

a. 5-ton cooling system
b. Duct work
c. Gas furnaces and evaporative coils
d. Installation – slab wiring

Example A. Continued.

Recurring costs:
a. Filters
b. Belts
c. Salaries of maintenance personnel
d. Cost of power

In the preceding example, notice that the one-time costs involve a single purchase, whereas the recurring costs could involve several purchases as time elapses, for example, as belts wear out they must be replaced, fuel bills must be paid every month, and so forth. It is important to know how often a cost will have to be paid – once or more than once – to get an idea of total cost.

ECONOMIC EFFICIENCY

The laws of thermodynamics state that there is a limit to the amount of heat that can be transferred or work that can be done by a quantity of energy. How well a system uses energy is its efficiency. One thermodynamic definition of efficiency is given in Equation 2 below:

$$E = \frac{\text{Energy transfer (of a desired kind) achieved by a device or system}}{\text{Energy input to the device or system}} \qquad \text{Equation 2}$$

where:

E = The efficiency of the device or system.

The more efficient a system is in performing its function, the less energy it uses; therefore, one way to reduce energy use is to increase energy efficiency. Combustion processes in industry are sometimes quite inefficient, and corrective measures, such as installation of a microcomputer system to monitor combustion, can increase efficiency.

The concept of efficiency is extended to energy economics as a simple extension of thermodynamic efficiency (which accounts for the cost of energy). The definition of economic efficiency is illustrated in the following equation:

$$E_{econ} = \frac{\text{Energy transfer (of a desired kind) achieved by a device or system}}{\text{Dollar cost of energy input to the device or system}} \qquad \text{Equation 3}$$

where:

E_{econ} = The economic efficiency of a device or system.

The difference between thermodynamic efficiency and economic efficiency is important. Consider two heating systems: one that is powered by natural gas and another by heating oil. The thermodynamic efficiency of the two systems might be equal (that is, the amount of heating is accomplished for each Btu of energy input); but if one fuel costs more than the other, the economic efficiencies would be different. When all variables are equal (such as availability of fuel) the system with the highest economic efficiency should be used. If two systems use the same kind of energy, then the system that is more thermodynamically efficient will be more economically efficient.

DETERMINANTS OF PRICE

When considering the costs of an energy-using system or of an energy conservation project such as insulating a home, it is very useful to know how costs are determined. Any cost is also a price - the price of whatever a business or individual buys with respect to the project or system. In the United States, prices are determined in free markets. A free market is a situation where people who want to buy something and people who want to sell the same thing communicate with each other and bargain about the price. The price agreed upon is the price of the product. In most markets there are a large number of buyers and sellers, and a great deal of "shopping around" usually takes place. The bargaining process can be seen in two forces: (1) the supply of a product by the people who want to sell it; and (2) the demand for a product by people who want to buy it.

To make the explanation of supply and demand easier to understand, one situation will be used throughout. This situation is the supply of and demand for gasoline in the United States.

SUPPLY

The supply of a product is a function of how much producers of the product will sell at each of a number of different prices. In most cases, people will not want to sell the same amount of something if its price changes. The relationship between price and the amount people will offer for sale (the quantity supplied) is a direct one. People will want to sell more at higher prices; in other words, a higher price results in a higher quantity supplied.

To explain supply adequately, the concept of resources must first be presented. A resource is a commodity used in any part of production. Iron is a resource used to produce steel, as are the services of the people who work in the steel plant. Resources cost money; therefore, businesses want to make the best use of them they can.

The idea of supply is directly related to the use of the resource. Resources are used in any kind of production where something is produced for sale. In the production of gasoline, the resources used include labor, management services, crude oil, chemical agents used in the refining process, and others. As the price of gasoline increases, producers can make more money by using more resources to produce gasoline. On the other hand, if the price of gasoline decreases, people will withdraw resources from gasoline production and use them to produce something else. The more resources used, the more gasoline produced. A direct relationship exists between the price of a good or service (in this example, gasoline) and the quantity of that good or service supplied by the producers. This relationship becomes quite clear when it is presented graphically as a supply curve (Figure 1).

By examination of the graph in Figure 1, one is able to discern that Gulf Corporation will supply 250 gallons of gasoline per day at a price of 70 cents per gallon. However, if the price rises to $1, then Gulf will increase the quantity of gasoline supplied to 400 gallons per day. Since 400 is greater than 250, a direct relationship between price and quantity supplied is shown to exist.

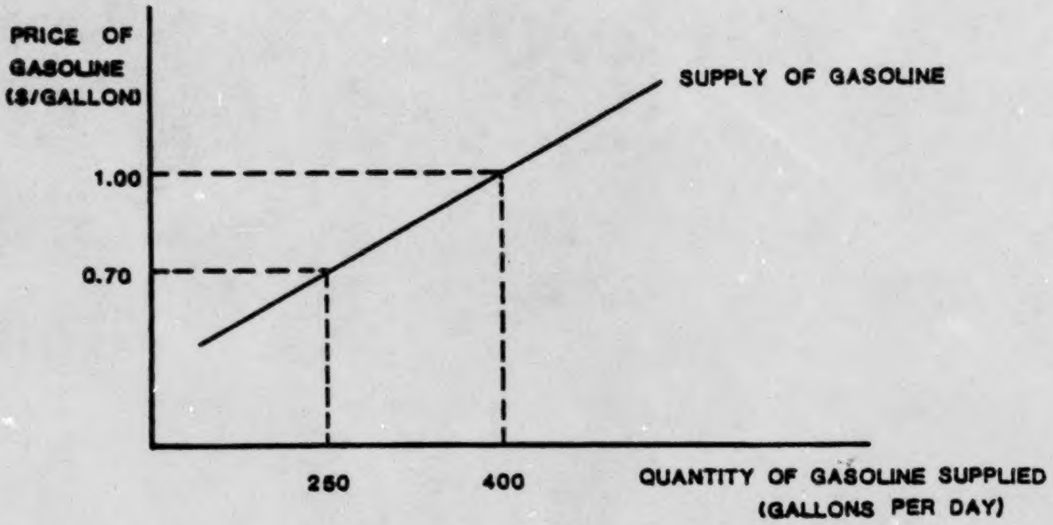

Figure 1. Supply Curve for Gulf Oil Corporation.

The concept of supply does not apply only to oil companies, but, rather, to all producers. Westinghouse will offer more 5-horsepower electric motors for sale if the price of the motors increases. The quantity of belts supplied by a parts manufacturer will decrease if the price of the belts decreases. Since this relationship applies for all products, the following law can be stated:

LAW OF SUPPLY: The higher the price of a product, the more of that product a business will offer for sale.

MARKET SUPPLY

The preceding description of supply only pertains to one business selling a particular product; but, the market consists of several companies that are trying to sell this product. Each of the many oil companies will want to sell a certain amount of gasoline at each particular price. All that is required to determine the market supply (the total amount of gasoline that will be for sale in the market at each of a set of prices) is a simple technique of addition. For each price the market supply is the sum of the quantity supplied by each seller in the market. For example, suppose there are only two producers of gasoline in the market — Gulf Corporation and Texaco Corporation; then the market supply curve is just the horizontal addition of the supply curves of the two corporations. This process is illustrated in Figure 2. At a price of 70 cents, Texaco Corporation will supply 200 gallons and Gulf Corporation will supply 150 gallons. Then the total supply in the market is (200 + 150), which is equal to 350 gallons on the market-supply graph shown. Similarly, at the higher price of $1, Texaco Corporation will supply 350 gallons and Gulf Corporation will supply 250 gallons. Thus the market supply at the price of $1 is (350 + 250) gallons, which equals 600 on the market supply curve. Notice that the direct relationship

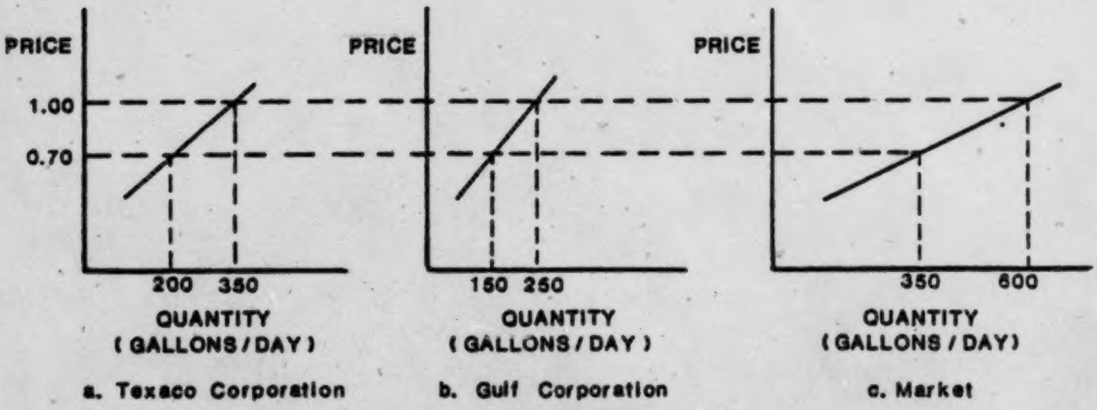

Figure 2. Derivation of the Market-Supply Curve. (Two Sellers in the Market).

that existed between price and quantity supplied in the case of the individual firm also applies to the market. In most cases the market-supply curve will be the main area of interest with regard to the determination of the price of a product. This type of horizontal addition also can be applied when there are more than two sellers in the market. The process is exactly the same: The quantity supplied by each firm is added at each price, and the result is the market supply.

DEMAND

The demand for a product is established by the quantity people want to buy at a specific set of prices. Like the supply curve, the demand curve is a set of possible alternatives. An individual who purchases a certain amount of gasoline when it is selling at 70 cents per gallon should be expected to buy less gasoline at $1 per gallon, all other things being equal. Therefore, an inverse relationship exists between the price of gasoline and the quantity of it demanded by a consumer.

Demand is related to the usefulness of a product, as compared to its price. At a given price a consumer finds it worthwhile to spend money for a certain amount of gasoline; but if the price increases, the consumer cannot justify spending the extra money needed to maintain the old level of use. Thus, the consumer reduces the level of gasoline usage and increases the surplus money, which can be used for something else. For this reason, an inverse relationship exists between the price of gasoline and the quantity of gasoline demanded by a consumer. This inverse relationship can be seen by examining the graphical representation of one consumer's demand curve, as shown in Figure 3. This curve shows the quantity of gasoline demanded by one consumer at each of many possible prices.

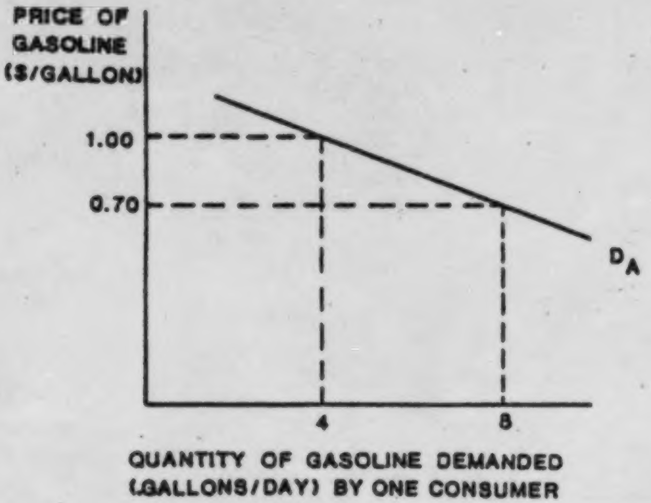

Figure 3.
Demand Curve
of Consumer A.

Notice that at a price of 70 cents per gallon the consumer demands 8 gallons per day. But, when the price increases to $1 per gallon, the quantity demanded by consumer A falls from 8 gallons to 4 gallons. There is an inverse relationship because a higher price results in a lower quantity of gasoline demanded.

It is important to realize, of course, that the concept of demand does not apply just to gasoline, but to all products. Similarly, as the price of energy increases, businesses want to buy less of it, and the role of the energy specialist is to find ways to reduce energy usage.

MARKET DEMAND

In most cases, and especially in the case of the gasoline market, many consumers want to buy the product. Thus, the need arises for the determination of market demand – the total amount all possible buyers will want to buy at each possible price. Market demand is derived in the same manner that market supply is derived: The sum of the quantities demanded by each consumer at each possible price is taken to give the total quantity demanded in the market at those prices. Graphically, market demand is simply the horizontal sum of all individual consumer-demand curves.

For simplification, assume that the market for gasoline has only two possible buyers: consumer A and consumer B. Now observe the following graphs in Figure 4.

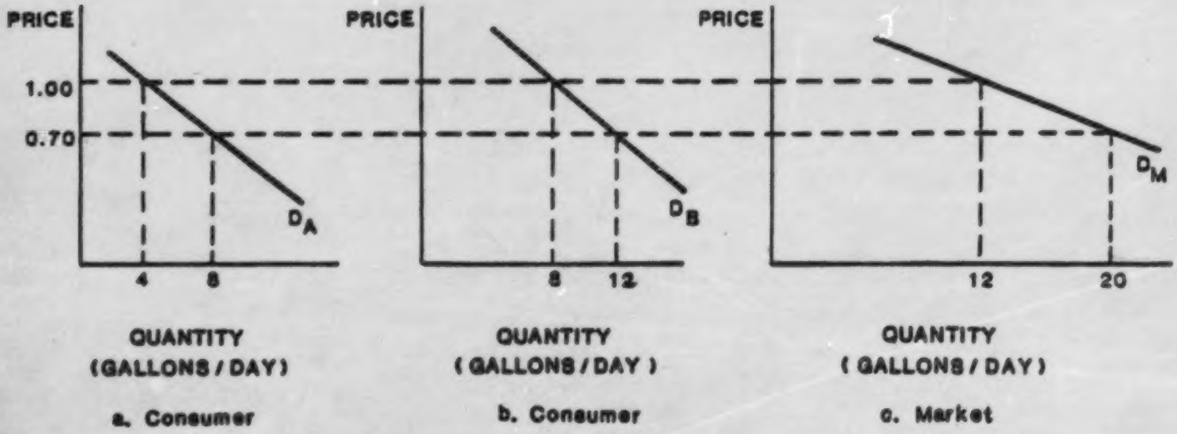

Figure 4. Derivation of the Market-Demand Curve. (Two Buyers in the Market.)

At a price of 70 cents, consumer A demands a quantity of 8 gallons per day, and consumer B demands a quantity of

12 gallons per day. Thus, the total demand in the market at the price of 70 cents is 8 gallons + 12 gallons, shown on the market-demand curve as 20 gallons per day. Similarly, at a price of $1, the quantity demanded by consumer A decreases to 4 gallons per day, and the quantity demanded by consumer B decreases to 8 gallons per day. Thus, the market demand decreases to 4 gallons + 8 gallons = 12 gallons per day. Notice that the inverse relationship that existed between price and quantity demanded in the case of the individual consumer also is applicable for the market as a whole. In most cases, the main area of interest with regard to the determination of the price of the product will be the market-demand curve. Of course, this process is applied just as easily when there are more than two possible buyers in the market; in this case, the quantity demanded by all consumers at each possible price is summed.

MARKET PRICE

The price that is charged for a product — the market price — is determined by the interaction of market supply and market demand. Market supply indicates how much of a product producers will offer for sale at each possible price, and market demand indicates how much of a product consumers will buy at each possible price. A sale will occur when a buyer and seller come to an agreement regarding a particular price for a particular amount. The process by which this agreement is obtained is illustrated by the graph shown in Figure 5. In this graph the market-supply curve and the market-demand curve are represented together.

At a price of $2 per gallon, producers are willing to supply 600 million gallons of gasoline per day, but consumers are willing to buy only 200 million gallons per day. Since the amount of 600 million gallons is much greater than 200 million gallons, the buyers and sellers in this case do not agree about price and quantity. At a price of 65 cents, producers are willing to supply only 250 million gallons per day, whereas consumers will demand 500 million gallons per day. Since 250 million gallons is considerably less than 500 million gallons, again it can be assumed that consumers and producers do not agree about price and quantity.

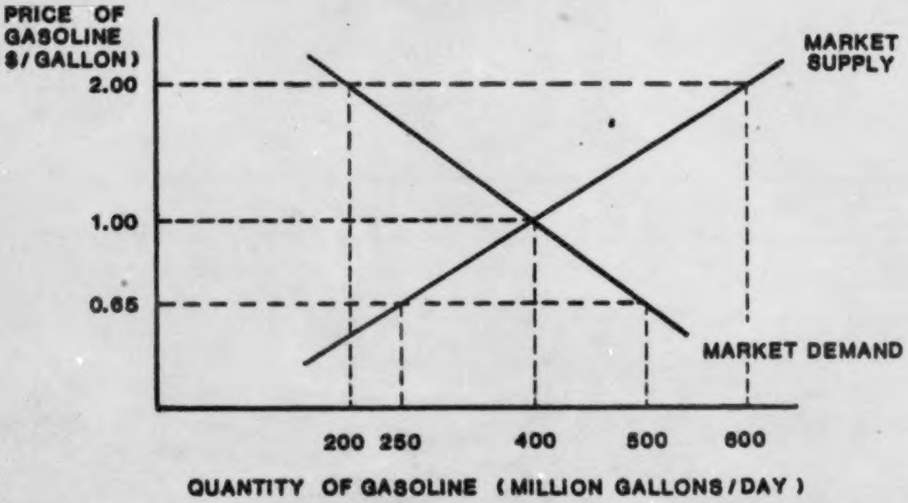

Figure 5. Determination of Market Price of Gasoline.

If the price of gasoline were $2 per gallon, producers would have an amount of gasoline equal to 600 – 200 million gallons that they could not sell. As a result, they would lower their price to try to sell it. However, if the price of gasoline were 65 cents per gallon, consumers would demand an amount equal to 500 — 250 million more than producers would be willing to sell at that price. As a result, producers would raise their price since people would buy all they offer for sale. The market price of gasoline is the price at which consumers are willing to buy exactly the same amount as producers are willing to sell — that is, market supply is equal to market deamnd. In Figure 5, market supply is equal to market demand at a price of $1 per gallon; at this price, producers supply the same quantity as consumers demand — the quantity of 400 million gallons per day. In this market, the price will always work toward $1, and the quantity sold will always work toward 400 million gallons. This price and quantity will not change unless some other factors have an effect on the market.

EXAMPLE B: DETERMINATION OF PRICE.

Given: Figure 6 shows the market-demand curve and market-supply curve for natural gas.

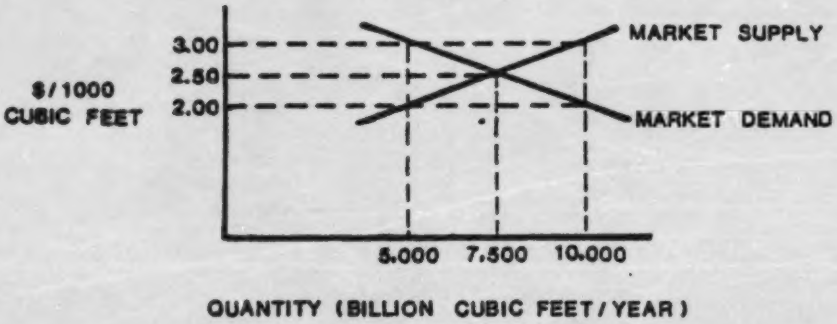

Figure 6. Supply and Demand of Natural Gas.

Example B. Continued.

Find: The market price of natural gas.

Solution: The market price occurs where quantity supplied is equal to quantity demanded — which, in the above figure occurs at 7,500 billion ft^3 (cubic feet) per year. The price at this quantity is $2.50 per thousand cubic feet. This is the market price.

ADDITIONAL FACTORS THAT INFLUENCE PRICE

The preceding analyses assume that the market is completely free — that no outside forces interfere with the choices of producers and buyers. Unfortunately, this situation is not always the case. External forces usually do have the effect of changing the quantity supplied at each price and/or the quantity demanded at each price. In the market for gasoline, there is a particularly strong influence: the policy of the Organization of Petroleum Exporting Countries (OPEC). This policy causes gasoline prices to be higher than they otherwise would be. Exactly how OPEC does this will be considered at this time.

The members of OPEC are suppliers of crude oil, the substance from which gasoline is refined. Thus, the supply curve of the OPEC countries is part of the overall supply curve of gasoline. OPEC restricts supply and increases gasoline prices in the following manner: Since the demand for crude oil is very high, and these countries can, in effect, sell all they produce, they simply reduce the amount they will produce at every possible price. This causes consumers to pay more for the gasoline they must have, especially since many consumers feel they cannot cut back on gasoline consumption - even if it costs more. The above process is demonstrated graphically in Figure 7.

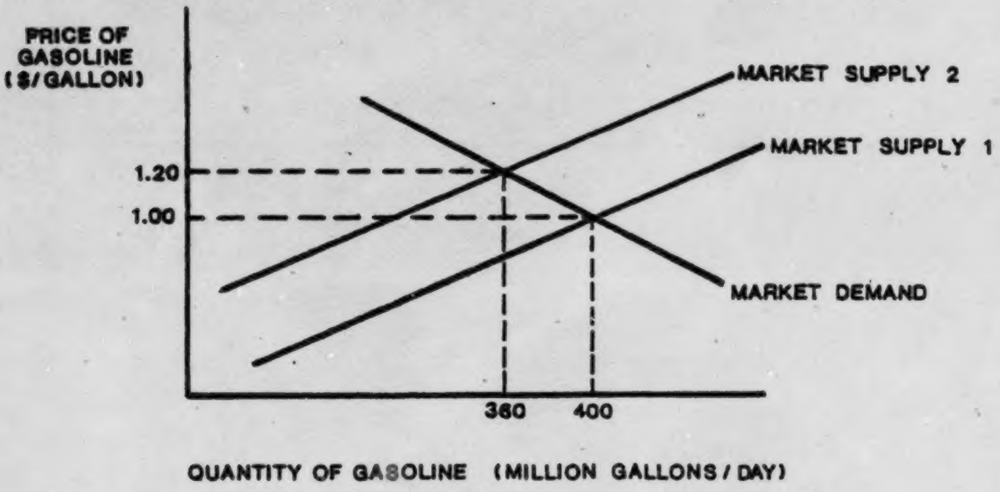

Figure 7. How OPEC (Members) Raise the Price of Gasoline.

Assume that before OPEC was formed the market price of gasoline was $1, with an accompanying quantity of 400 million gallons per day. This means that the original supply curve was market supply 1, since it intersects with market demand at those points. However, when OPEC was formed, the effect of refusing to supply as much crude oil at each price as before was to shift the market-supply curve from market supply 1 to market supply 2 (Figure 7). Less gasoline was offered by producers at each price. Since the demand schedule did not change, the point of intersection of market supply and market demand is at a higher prices ($1.20) and a lower quantity sold (360 million gallons per day); therefore, the effect of OPEC's action is a higher price and a lower quantity sold. Since prices rise more than quantity sold declines, the OPEC nations make more money.

Another factor that can influence supply and demand is the price of a related good or product. This is particularly true with regard to different forms of energy. If the price of coal increases, then some industrial plants might change their production process to diesel fuel. This would increase the demand for diesel fuel at each price, and the increased demand would lead to an increase in price since the supply and demand curves would intersect at a higher point.

In summary, the following factors will result in a price increase:

- A decrease in supply
- An increase in demand
- A decrease in supply greater than a decrease in demand
- An increase in supply greater than an increase in demand

On the other hand, the following factors will result in a lower price:

- An increase in supply
- A decrease in demand
- An increase in supply greater than an increase in demand
- A decrease in supply greater than a decrease in supply

These conclusions are easy to understand if the graphical representation of supply and demand, and where they intersect, is considered in each case. The role of the energy specialist is to help reduce the demand for energy. This, in turn, would reduce the price of energy – or, at least, it would keep price increases smaller.

MARGINAL ANALYSIS

Most energy conservation projects involve costs of one kind or another. However, because these projects save energy, thereby saving money, they also involve cost savings. Naturally, an energy project should save at least as much as it costs. Marginal analysis can be used to determine how much should be spent on energy conservation, because it shows how to save the most money. The use of marginal analysis is also a good way to compare energy projects that do not cost the same amount of money to implement.

MARGINAL COST AND MARGINAL COST SAVINGS

Marginal refers to the last increment of some variable, such as the last inch of insulation in an attic or the last solar panel installed on the roof of a building. Marginal cost (MC) is the cost of adding the last unit. Marginal cost savings, or simply marginal savings (MS), are the dollar savings that result from adding the last unit. Notice that these figures apply only to the last unit and not the total system or project. Example C illustrates how to calculate marginal cost.

EXAMPLE C: DETERMINATION OF MARGINAL COST.

Given: A solar equipment supplier will supply a 30-square-foot solar panel for $500. However, if five or more panels are bought, the price decreases to $480 per panel.

Find: The marginal cost of the fifth panel.

Solution:
Cost of 4 panels = 4 x $500 = $2,000.
Cost of 5 panels = 5 x $480 = $2,400.
Marginal cost of fifth panel = Cost of 5 panels - Cost of 4 panels
= $2,400 - $2,000
= $400.

HOW TO USE MARGINAL ANALYSIS

It is obvious to the energy specialist that expenditures are economically feasible as long as cost savings are greater than costs. With respect to marginal analysis, additional units should be added as long as the marginal cost of the unit is less than the marginal cost savings of the unit. This will occur up to the point where the marginal cost savings are equal to the marginal cost. Cost savings will decline as more and more units are added; and, eventually, they will approach the marginal cost of adding extra units. As long as the marginal savings exceed the marginal cost, it is economically sound to add another unit. Only when the marginal cost becomes equal to the marginal savings will adding another unit lose money. The process of comparing marginal cost and marginal cost savings is shown in Example D.

EXAMPLE D: MARGINAL ANALYSIS AND THE PROPER AMOUNT OF INSULATION.

Given: The marginal cost and cost savings of insulating the attic of a home with different thicknesses of insulation are as follows:

Amount of Insulation (in inches)	Marginal cost per sq ft of last inch	Marginal cost savings per sq ft of last inch
1	$0.07	$1.50
2	0.02	0.50
3	0.02	0.20
4	0.02	0.10
5	0.02	0.07
6	0.02	0.05
7	0.02	0.03
8	0.02	0.025
9	0.02	0.015

Find: The proper amount of insulation to install.

Solution: The proper amount is where marginal cost is equal to marginal cost savings. This occurs between the eighth and ninth inch of insulation. Therefore, if the homeowner installs the eighth inch, the cost savings are greater than the costs. However, if the ninth inch is installed, the cost of that inch is greater than the cost savings it generates. Since insulation cannot be bought by the half-inch the least profitable inch to install in this case is the eighth inch. Therefore eight inches of insulation should be installed.

When all energy conservation opportunities have been implemented up to the point where the marginal cost of the last unit is equal to its marginal cost savings, then energy conservation past that point would be unprofitable.

Marginal analysis normally is applicable when money which is to be spent on energy conservation can be spent on various amounts of some product, such as insulation or solar collectors. To achieve maximum accuracy, marginal analysis should be applied to the smallest possible units. For instance, it would be better to analyze solar collectors by the square foot than by some larger collector size — such as 32 ft^2. Economic principles can be applied to energy use and conservation because energy costs money. Not all techniques are relevant in all situations, but every situation will have costs and/or cost savings that can be analyzed using the principles of energy economics. A knowledge of these principles gives the energy specialist the ability to determine whether a project should be implemented and to what extent. This module has presented some background materials which can be useful in this process.

EXERCISES

1. Supply and Demand

At a price of \$1.75/1,000 ft^3 of natural gas, five companies in the State of Texas will supply 10 million ft^3 per day. If the price decreased to \$1.25/1,000 ft^3, the companies would sell 3 million ft^3/day. If the price increased to \$2.25/1000 ft^3, the companies would sell 17 million ft^3/day. Plot these prices and quantities supplied on the graph in Figure 8. Connect the points with a line, herein referred to as a supply curve.

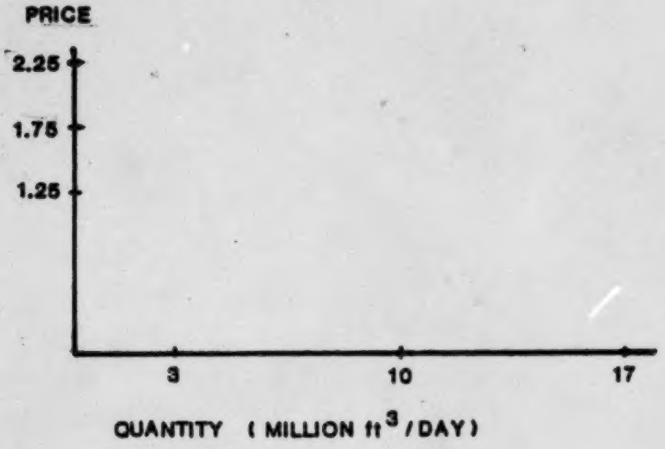

Figure 8. Graph for Plotting Supply and Demand Curves.

The amount of gas that businesses and individuals in the State of Texas will buy varies with price. Following is a demand schedule showing the quantities of gas demands at various prices. Plot the demand schedule on the graph in Figure 8. Connect the observation to form a demand curve.

Price	Quantity demanded (10^6 ft^3/day)
\$2.25	3
1.75	10.5
1.25	18

a. What will be the price and quantity of the natural gas in Texas?

If a new natural gas field is discovered by one of the companies, the quantity of gas supplied at every price would increase to the following:

Price	Quantity supplied (2) (10^6 ft^3/day)
\$2.25	21
1.75	14
1.25	7

Plot the new supply curve.

b. The supply curve has shifted to the (right, left).

c. The shift in the supply curve will cause the price of gas in Texas to (increase, decrease) to ________________ (what level).

d. The shift in the supply curve will cause the market quantity of gas to (increase, decrease) to ________________ (what level).

e. Discovery of new resources (increasing supply) can be expected to have what effect on the cost of energy?

2. Marginal Analysis

Fowl Foods is an independent, turkey-processing plant. The plant has four refrigeration rooms in which it can store up to 800,000 pounds of processed turkeys. These rooms are currently kept at a temperature of 27.5°F. The energy specialist is confident that Fowl Foods can substantially save on its utility bills by raising the temperature in the refrigeration units. However, there is a question unanswered: To what degree should the temperature be raised?

By considering the type of cooling system, the cubic feet of air in the room, the average climate condition of the region, and the cost of fuel, it has been determined that Fowl Foods can save \$4,000 ($\sqrt{\text{degrees raised}}$) per year, per degree the temperature is raised above 27.5°F. However, the quality control department maintains that the rate of spoilage of the turkeys will increase with the warmer temperature. Based on historical data, the rate of spoilage can be calculated by the following formula:

$$\text{Annual \# spoiled turkeys} = 1{,}215 + (\text{Temperature} - 28°\text{F})^2(2{,}000)$$

At 27.5°F, therefore the spoilage equals

$$1{,}215 + (28.5°\text{F} - 28°\text{F})^2(2{,}000)$$

or,

$$1{,}215 + (0.5°\text{F})^2(2{,}000)$$

or,

$$1{,}215 + 500 = 1{,}715 \text{ turkeys per year.}$$

a. At 29°F, the spoilage equals

$$\begin{aligned}\text{Spoilage} &= 1{,}215 + (29 - 28)^2(2{,}000)\\ &= 1{,}215 + (1)^2(2000)\\ &= 1{,}215 + 2{,}000\\ &= 3{,}215.\end{aligned}$$

The cost of each lost turkey is approximately \$2. The marginal cost of raising the temperature from 28.5°F to 29°F for Fowl Foods is (3,215) - (1,715) = (1,500)(\$2) = \$3,000.

New spoilage - old spoilage = (marginal spoilage) (cost/unit) = marginal cost

The cost savings from raising the temperature from 27.5°F to 29°F = $4{,}000\sqrt{1.5} = \$4{,}898.98$.

Since \$4,898.98 > \$3,000, the temperature should be raised to at least 29°F.

b. Calculate the spoilage at 30°F. What is the marginal spoilage over 29°F? What is the marginal cost of raising the temperature to 30°F?

Calculate the cost savings of raising the temperature from 29°F to 30°F. Should the temperature be raised to 30°F?

c. Calculate the spoilage at 31°F. What is the marginal spoilage over 30°F? What is the marginal cost of raising the temperature to 31°F?

Calculate the cost savings of raising the temperature from 30°F to 31°F. Should the temperature be raised to 31°F?

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GLOSSARY

Cost minimization: A method of increasing profits by minimizing the amount of cash a business must spend.

Demand: A quantity established by customers who want and are able to buy a product at specific set prices.

Demand-price: The relationship between how much of a product consumers want to buy and the price of that product.

Direct cost: A cost that involves the actual spending of money.

Economic efficiency: The energy transfer achieved by a device or system, divided by the dollar cost of energy input to the device or system.

Efficiency: How economically and effectively a system uses energy.

Energy Economics: The dollar comparison between various energy conservation alternatives to determine the extent of energy savings (reduced costs) that each alternative will produce in relation to the dollars expended to put the alternative in place.

Free market: A situation where consumers who want to buy a product and producers who want to sell that product communicate with each other and bargain about the price of that product.

Indirect cost: A cost that is incurred when a business or individual performs a task rather than paying to have it performed. It is the amount of money that could have been earned during the time it took to perform the task.

Marginal analysis: A technique that emphasizes the incremental benefit and cost of an additional project or decision.

Marginal cost: The cost of adding the last unit to an energy system.

Marginal cost savings: The dollar savings that result from adding the last unit to an energy system.

Market demand: The total amount all possible buyers will want to buy at each possible price. It is the sum of all individual demands.

Market price: The price at which consumers are willing to buy exactly the amount the producers are willing to sell. It is determined by the intersection of the supply and demand curves.

Market supply: The total amount of a product that will be for sale in the market at each of a set of prices.

Profit: The amount of money paid to a business for the goods or services it sells (total revenue), minus the amount of money the business must pay out in operating expenses (total cost).

Profit maximization: Achieving the greatest difference between total revenue and total cost.

Recurring cost: Costs that are paid on a regular basis over a specified period of time.

Resource: A commodity used in any part of production.

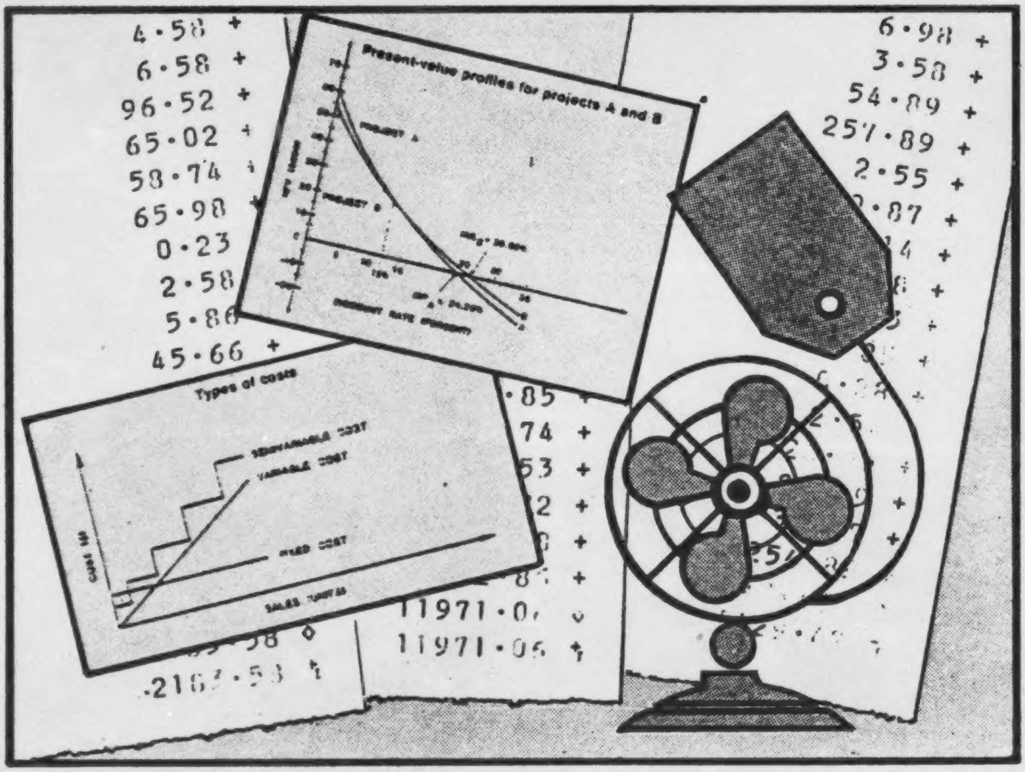
Supply: How much of a product producers will sell at each of a number of different prices.

Supply curve: The graphic representation of the direct relationship that exists between the price of a good or service and the quantity of that good or service supplied by the producers.

ENERGY TECHNOLOGY

CONSERVATION AND USE

ENERGY ECONOMICS



EE-02

FINANCIAL PARAMETERS OF ENERGY ECONOMICS

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4800 LAKEWOOD DRIVE, SUITE 5
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INTRODUCTION

This module introduces the student to several fundamental techniques used to analyze the costs and cost savings of energy projects. These techniques enable the student to understand the important relationship that exists between time and money and why this relationship must be considered if costs and cost savings are to be analyzed accurately.

PREREQUISITES

The student should have a good understanding of basic algebraic functions and should have completed Module EE-01 of Energy Economics.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Define the following terms:
    a. Principal.
    b. Interest.
    c. Interest rate.
    d. Time period.
    e. Future value.
    f. Compounding.
    g. Cost escalation.
    h. Present value.
    i. Discount rate.
2. Explain the time value of money and why it must be considered in the analysis of the costs and cost

savings of energy projects.

3. Know when to use the following:
   a. Future value interest factor.
   b. Present value interest factor.
4. Given the proper information, compute the following:
   a. The future value of any amount.
   b. The present value of any amount.
5. Give and explain the information needed to calculate cost escalation.
6. State the relationship between future value and the following:
   a. Interest rate.
   b. Number of time periods.
7. State the relationship between present value and the following:
   a. Interest rate.
   b. Number of time periods.
8. Read, interpret, and use the following:
   a. The Future Value of $1 Table.
   b. The Present Value of $1 Table.
9. Calculate future values and present values in situations where time periods shorter than one year are involved.

PRINCIPAL AND INTEREST

Before money is considered for investment in energy conservation measures, it is important to consider what occurs when money is deposited in a bank account. The basic concepts of this are understood more easily when the following terms are defined:

- Principal: The amount of money necessary to replace, install, or modify an existing energy system. It is always a dollar amount. Principal can be calculated by answering the following question: If a business has no money of its own, how much would it have to borrow from a bank to replace, install, or modify the system?
- Interest: A dollar amount paid to the lender from the borrower for the use of the lender's money.
- Interest rate: A percentage figure, usually between 5 and 20, that indicates the portion of the principal that must be paid in interest over each time period. A 10% interest rate on a $1000 loan means the borrower must pay 10% of $1000 (10% of $1000 = 0.10 x $1000 = $100) for the use of the money for each time period until the $1000 is repaid. Equation 1 can be used to determine the amount of interest a borrower will pay a lender after any 1-year period. (Interest and interest rates are further illustrated in Example A.)

$$\text{Amount of interest paid after any 1 year} = \text{Amount in account at the start of the year} \times \text{Interest rate} \qquad \text{Equation 1}$$

- Time period: The length of time with which the interest rate is associated. In most cases, the time period is expressed in years – the indicated percentage of the principal must be paid for each year the principal is borrowed. When time periods other than years are used, the appropriate interest rate figure is annual interest rate divided by the number of time periods per year. For example, if the annual interest rate is 18% and one is trying to calculate monthly interest the relevant interest rate is $\frac{18\%}{12} = 1.5\%$. The 12 in the denominator comes from the number of months in a year.

The relationship of the above defined terms is illustrated in Figure 1.

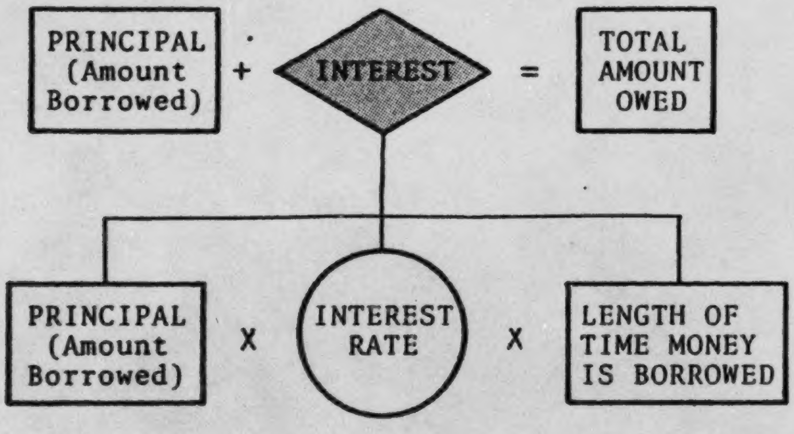
PRINCIPAL (Amount Borrowed)
+
INTEREST
=
TOTAL AMOUNT OWED
PRINCIPAL (Amount Borrowed)
X
INTEREST RATE
X
LENGTH OF TIME MONEY IS BORROWED

Figure 1. Relationship Among Principal Interest (Rate), and Time.

The definition of interest rate implies that the amount of interest associated with each year is the same. Futhermore, this amount is determined by the interest rate; the higher the rate, the higher the interest charge will be for each year the principal is borrowed.

Whenever money is placed in a certain kind of bank account, it draws interest. The money draws interest because the bank borrows the money from the person having the account. Therefore, when individuals or businesses have a sum of money that is not needed immediately, it may prove to be advantageous to deposit the money in a bank account that will allow the money to earn interest. The result, of course, is more money. To summarize, costs and cost savings from energy conservation projects should be analyzed as follows: If the amounts of money are to be kept over some period of time, or received after some period of time has elapsed, it is necessary to consider what would happen if the money were drawing interest instead of being invested in energy conservation measures. Depending on the interest rate, this method of examining costs and cost savings can have an important effect on decision making.

TIME VALUE OF MONEY

Consider the energy specialist who is debating the installion of plastic tint for all windows in a particular building. The building has 1500 square feet of glass area, and since the cost of installation would be \$1.50 per square foot, the total installation cost would be \$1.50 x 1500 = \$2,250. How much money (in the form of lower fuel bills because of reduced energy usage) must the plastic tint save the company to justify the \$2,250 installation cost? Obviously, the tint must save at least \$2,250. But what about the time periods between the initial cost and the reduction in fuel bills? Indeed, time is a very important factor in energy conservation decisions.

Suppose this procedure would save $225 per year for 10 years. Then the total savings would be $2,250, and the cost would equal the cost savings if the time element is ignored. The time element cannot be ignored, however, for the following reason: If the owners of the building were to take the $2,250 now and deposit it in the bank, then the entire $2,250 would immediately start earning interest (money). On the other hand, if the plastic tinting is installed, then only $225 can be deposited in the bank the first year to earn interest. Since $225 will be added each year (for a total of 10 years), the amount in the bank will increase to $450 the second year, $675 the third year, and so forth. Obviously, the owners would prefer to have $2,250 in the bank earning interest every year than some amount less than $2,250, since the amount of interest earned is a percentage of the amount in the account. Therefore, in this particular case, the installation of the plastic tinting is not justified.

The situation just described is an example of time value of money. A very distinct and persistent relationship exists between time and money. One dollar now is worth more than the guarantee of the receipt of $1 at some time in the future. This is true because of the following: One dollar now can be placed in the bank to earn interest. When the designated time in the future arrives, the interest earned will have resulted in a total amount greater than the original $1. If the interest rate is 6%, after one year, $1 will have an earned interest of 6 cents, and the resulting $1.06 is preferable to the guarantee of the receipt of just $1 one year from now. This is the difference between an amount of money now and the guarantee of the receipt of the same amount later. Therefore, interest provides the link between present dollars and future dollars. This relationship between time and money is illustrated in Figure 2 and Example A.

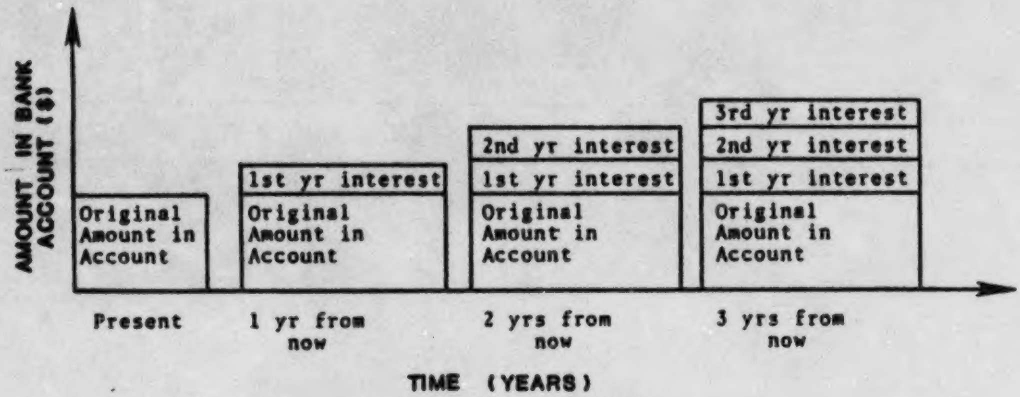

Figure 2. The Time Value of Money. (Accumulation of an Amount Placed in a Bank Account.)

Notice that for each year the bank keeps the money it must pay interest to the owner of the money. The amount of interest the bank must pay the lender depends on the rate of interest, principal, and time the money is deposited in the bank. The following equations (Equation 2) is offered as evidence:

$$I = prt \qquad \text{Equation 2}$$

where:

I = Interest.
p = Principal.

r = Rate.

t = Time.

In most situations, it is important to know the exact amount of money that will accumulate in the bank account. Example A will explore this question.

EXAMPLE A: INTEREST AND INTEREST RATES.

Given: Fifty dollars is deposited in a bank account.

Find: The amount of interest that would be paid on the $50 each year if the interest rate were as follows:

a. 5%
b. 10%
c. 15%

Solution: An interest rate can be expressed as a decimal figure for use in calculations. For 5%, use 0.05; for 10%, use 0.10; and for 15%, use 0.15. Equation 1 can be used as follows:

a. $$\text{Interest paid after 1 year} = \text{Amount in account at the start of the year} \times \text{Interest rate}$$

$$= \$50 \times 0.05$$

$$= \$2.50$$

b. $$\text{Interest paid after 1 year} = \$50 \times 0.10$$

$$= \$5.00.$$

c. $$\text{Interest paid after 1 year} = \$50 \times 0.15$$

$$= \$7.50.$$

FUTURE VALUE OF A FIXED AMOUNT

The future value of an amount of money is the dollar figure the amount will become if placed in a bank account with a particular interest rate for a certain time period (usually years). Consider the following situation where $500 is deposited in a bank account that pays 7% interest per year, and knowledge is needed of the amount that will be in the account after 4 years. The accumulation of the $500 proceeds as follows:

- The first year, the account will draw interest equal to $500 x 0.07 = $35. Therefore, at the end of the first year, if this interest were left in the account, the account would contain $500 + $35 = $535.
- The second year, the account would draw interest on this $535, and the interest drawn would equal $535 x 0.07 = $37.45. Therefore, at the end of the second year, the account would contain $535 + $37.45 = $572.45.
- The third year, the account would draw interest equal to $572.45 x 0.07 = $40.07. Therefore, at the end of the third year, the account would contain $572.45 + $40.07 = $612.52.
- The fourth year, the account would draw interest equal to $612.52 x 0.07 = $42.88. Therefore, at the end of the fourth year, the account would contain $612.52 + $42.88 = $655.40.

The bank account that begins at $500 will accumulate to $655.40 in 4 years. When large amounts of money are involved, the time aspect becomes more important. In energy economics, the time value of money is an important consideration in the decision-making process.

COMPOUNDING PROCESS

In the preceding example, each time the interest was earned, it was left in the bank. In this way, interest earned in 1 year could itself earn interest the next year. This is called compounding. Compounding is the process wherein the interest earned on an amount of money earns interest itself in the following time periods. If the compounding process is allowed to continue each time period, the sum will accumulate much faster. Compounding will not occur if the interest earned each time period is removed from the account as it is earned – in this case the amount in the account will remain constant. The procedure – with regard to calculations made for energy-project decisions – is to treat each amount as if compounding would occur. This provides a more accurate reflection of the time value of money.

EXAMPLE B: THE COMPOUNDING OF AN AMOUNT OF MONEY.

Given: The owner of a building sells a used compressor for $200 and deposits the money in a bank account. The interest rate is 8%.

Find: The amount of money in the account after 3 years if the interest earned is not removed from the account.

Example B. Continued.

Solution: Recall Equation 1, and use it for all 3 years:

Interest paid after first year = Amount in account at the start of the first year x Interest rate

= $200 x 0.08

= $16.

Therefore, the amount in the account after 1 year is equal to $200 + $16 = $216.

Interest paid after second year = Amount in account at the start of the second year x Interest rate

= $216. x 0.08

= $17.28.

Therefore, the amount in the account after the second year is equal to $216 + $17.28 = $233.28.

Interest paid after third year = Amount in account at the start of the third year x Interest rate

= $233.28 x 0.08

= $18.66.

Therefore, the amount in the account after the third year is equal to $233.28 + $18.66 = $251.94.

FUTURE VALUE OF $1

With respect to the future value of an amount, consider what happened to the original $200 in Example B. First, the $20 was multiplied by the interest rate (8% = 0.08). This product was then added to the original amount, and the sum at the end of the first year was obtained. The amount present at the end of the first year was multiplied by the interest rate and then added to the amount which was present at the end of the first year. Thus, the amount present at the end of the second year was obtained. The same process was used for the third year.

The process wherein an amount is multiplied by some number and that product is added to the original amount is the process which is repeated when an amount is compounded. The first year of the preceding process is given by (\$200 x 0.08) + \$200. Now observe the following mathematical manipulations:

$$\begin{aligned}(\$200 \times 0.08) + \$200 &= (\$200 \times 0.08) + (\$200 \times 1)\\ &= (\$200 \times (0.08 + 1)\\ &= (\$200) \times (1.08).\end{aligned}$$

The result here is that the original amount is multiplied by a number that is equal to one plus the interest rate $(1 + r)$. This principle can be expressed as follows:

> Given an amount in a bank account at a certain time period, the amount in the account 1 year later is equal to the product of the original amount and a number equal to one plus the rate of interest $(1 + r)$.

This principle applies not only for the first year, but for each year thereafter. Since the amount at the end of the first year is equal to (\$200 x 1.08), the amount at the end of the second year would be equal to (\$200 x 1.08) x (1.08). This process is illustrated in Figure 3.

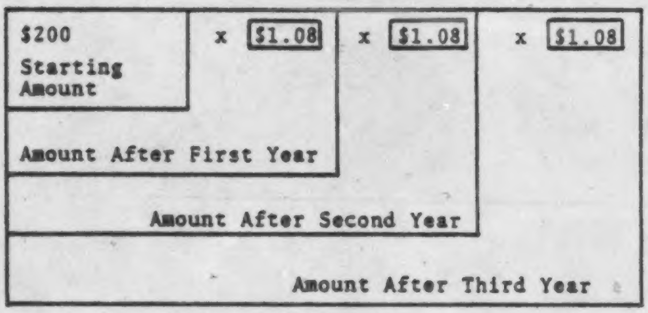

Figure 3. The Compounding of $200 for 3 Years with Interest Rate 8%.

Now consider the case where $1 is deposited in a bank account that earns interest at a particular rate. (By using the letter r for rate, the formula obtained will be applicable for all rates of interest.) The principle stated previously still applies, although the multiplication is much simpler since the original amount is $1. Figure 4 is similar to Figure 3, although the original amount is $1 and the interest rate is expressed as r.

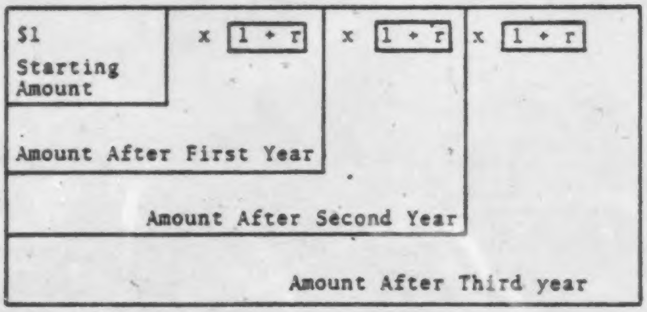

Figure 4. The Compounding of $1 for 3 Years with Interest Rate r.

It should be clear that the dollar amount present after 1 year is simply $(1 + r)$. Similarly, the amount present after 2 years is $(1 + r) \times (1 + r) = (1 = r)^2$. Note that this process can continue beyond 3 years for any length of time. All that is required is to multiply the amount in the beginning of last period by $(1 + r)$. In the special case of \$1, the amount in the account after a number of time periods (n) is equal to $(1 + r)^n$. Thus, the first formula is as follows:

$$FV_n(\$1) = (1 + r)^n \qquad \text{Equation 3}$$

where:

$FV_n(\$1)$ = Future value of a present amount \$1 after n time periods (usually years) in a bank account.

r = Interest rate.

n = Number of time periods the dollar accumulates.

EXAMPLE C: FUTURE VALUE OF $1.

Given: One dollar has been deposited in a bank account, earning 8% interest.

Find: The amount in the account after 3 years.

Solution:

$$\begin{aligned} FV_n(\$1) &= (1 + r)^n \\ &= (1 + 0.08)^3 \\ &= (1.08)^3 \\ &= \$1.26. \end{aligned}$$

FUTURE VALUE OF AMOUNTS GREATER THAN $1

When working with original amounts of more than $1 (which will normally be the case), consider the following: The process that has been described previously will occur with each dollar originally present. Therefore, all one must do is calculate the future value of $1 over the same time period, using the same interest rate. Then multiply this figure by the original amount to obtain the future value of that amount instead of $1. For instance, in Example C, if $50 had been deposited in the account, then each of the 50 dollars would have been worth $1.26 after 3 years. Therefore, the total future value of the $50 would be $50 x 1.26 = $63.

HOW TO READ THE FUTURE VALUE OF $1 TABLE

Table 1 gives the future value of $1 accumulated for a given number of time periods (years) at a given interest rate. This table, called Compound Sum of $1 Table, because of the compounding process that takes place, is used as follows:

- Find the appropriate interest-rate column.
- Find the appropriate time-period column.
- Find the entry where the row (time) and the column (interest) intersect.

This number is the accumulated value of $1 left in a bank account at the given interest rate for the given time period. Once this number is obtained, it can be multiplied by the original dollar amount to obtain the accumulated future dollar value of that amount. For this reason, an entry from the

TABLE 1. COMPOUND SUM OF $1.

Number of Periods (Time)	Rate of Interest			
	6%	7%	8%	9%
1	1.0600	1.0700	1.0800	1.0900
2	1.1236	1.1449	1.1664	1.1881
3	1.1910	1.2250	1.2597	1.2950
4	1.2625	1.3108	1.3605	1.4116

Compound Sum of $1 Table is often the Future Value Interest Factor (FVIF) for the particular interest rate and time period. (A number is said to be a factor if another number, or group of numbers, is multiplied by it.) The preceding examples of $50 placed in a bank account for 3 years, when the interest rate is 8%, will be worked now by using the preceding section of the Compound Sum of $1 Table.

The appropriate column to use is the one under 8%, and the appropriate row to use is the one labeled 3, since the period is 3 years. The entry wherein this row and this column intersect is 1.2597; thus, each dollar of the original $50 will accumulate to $1.2597. Therefore, the total amount in the account, after 3 years, is given by (1.2597) x $50 = $62.98. (Due to the rounding of numbers, this answer is slightly different from the one previously obtained.) With reference to Equation 2 and the preceding conclusion, the formula for the future value of a present amount ($P), which draws interest at a rate r for n time periods, can be given as follows:

$$FV_n(\$P) = \$P \times (1 + r)^n \qquad \text{Equation 4}$$

where:

$FV_n(\$P)$ = Future value of a present amount (\$P) after n time periods.
r = Interest rate.
n = Number of time periods the amount \$P accumulates at interest rate r.

Recall from Equation 3, however, that part of Equation 4 – $(1 + r)^n$ – is equal to the compound sum of \$1, which receives interest at a rate r for n time periods. This amount for different time periods and interest rates is what comprises the entries in the Compound Sum of \$1 Table. This amount is also the Future Value Interest Factor (FVIF) for the appropriate number of time periods and the appropriate interest rate. Therefore, Equation 4 can be rewritten as follows:

$$FV_n(\$P) = \$P \times FVIF\ (n\ year,\ r) \qquad \text{Equation 5}$$

where:

$FV_n(\$P)$ = Future value of a present amount \$P after that amount has accumulated in a bank account for a given number of years with a given interest rate.

FVIF (n years, r) = Future value of interest factor, or the future value of $1, if it accumulates for n years when the interest rate is 4% (the same n and r which apply to the amount SP).

EXAMPLE D: FUTURE VALUE OF AN AMOUNT GREATER THAN $1.

Given: An amount of $26,500 is placed in a bank account with an interest rate of 9%.

Find: The amount in the account after 4 years.

Solution: Recall Equation 4 as follows:

$$FV_4(\$26{,}500) = \$26{,}500 \times \text{FVIF (4 years, 9\%)}$$

From Table 1, FVIF (4 years, 9%) is obtained to be 1.4116. Therefore,

$$FV_4 = \$26{,}500 \times 1.4116$$
$$= \$37{,}407.40.$$

The FVIF method for determining future value involves much easier calculations and should be used whenever possible.

OBSERVATIONS CONCERNING THE FUTURE VALUE OF A FIXED AMOUNT

Close examination of the more complete Compound Sum of $1 Table at the end of this module (Data Table 1) will reveal two facts:

- Amounts accumulate faster at higher interest rates.
- Longer time periods result in greater accumulation.

The first fact applies because a higher interest rate means that a larger percentage of the original amount will be earned each year. Since the accumulation of 1 year will also earn interest in subsequent years, the total amount will accumulate faster at a higher rate of interest. The second fact applies because earnings from each additional year of interest add to the total sum. The relationships among rate of interest, accumulation, and time are given in Figure 5 and illustrated in Example E.

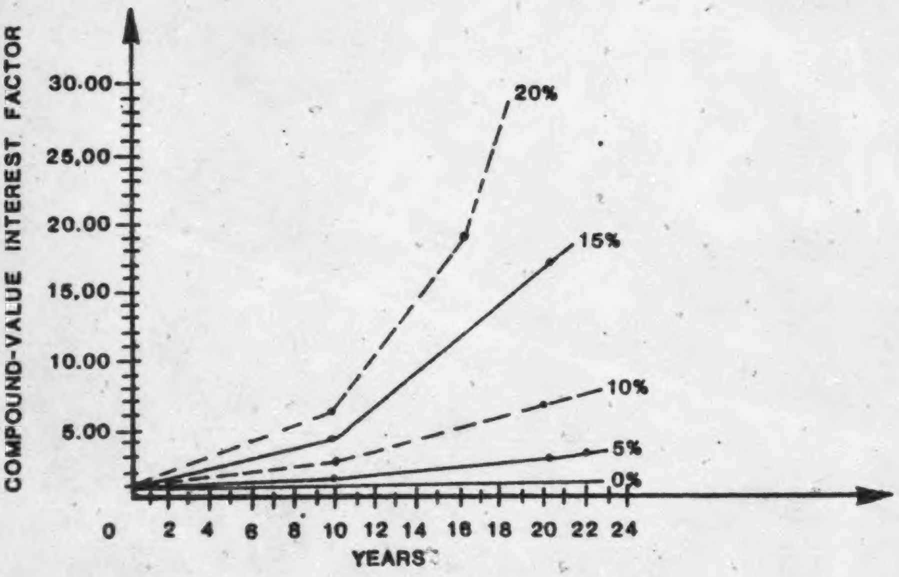

Figure 5. Future Value of $1.

EXAMPLE E: FUTURE VALUE OF A FIXED AMOUNT GREATER THAN $1.

Given: A business decides to buy two solar collector panels at $1000 each and has the option of paying $2000 now or $3000 5 years from now. The relevant interest rate is 10%.

Find: The payment plan that will minimize the cost the business must pay, using Data Table 1 at the back of this module.

Solution: The future value of $3000 to be paid in 5 years is $3000. The future value of $2000 present dollars, 5 years from now, is calculated as follows (by using Equation 5):

$$\begin{aligned} FV_5(\$2000) &= \$2000 \times \text{FVIF (5 years, 10\%)} \\ &= \$2000 \times 1.6105 \\ &= \$3221. \end{aligned}$$

FVIF (5 years, 10%) is obtained from Data Table 1. Thus, the future value of the $3000 payment in 5 years is less than the $2000 payment now, and the business could wait 5 years and pay $3000 to minimize the cost. Instead of paying $2000 now, the business could put the $2000 in a bank account drawing 10% interest per year. After 5 years, the business would have $3221 in the account and could pay the $3000 and use the remaining $221 for something else. If the $2000 is paid now, the business, in effect, loses $221.

COST ESCALATION

A very useful and important application of future value is the concept of cost escalation. Cost escalation is a method that considers the fact that the price of something will rise in the future. One area where cost escalation is most likely to occur is the price of energy. As shown in Table 1 of Module EE-01, prices of energy are rising, and all indications point to the possibility of their continued increase in the future. If the price of some form of energy (electricity, for example) is expected to increase a certain percentage each year, then the same thing happens to the price of that energy that happens to an amount of money that is deposited in a bank account drawing interest at an interest rate equal to that percentage. In both cases the original amount is increased by a constant percentage each year. Suppose the price of gasoline is 65 cents per gallon. If the price is expected to rise 15% during the next year, then the amount of the increase will equal $0.65 x 0.15 = $0.0975. Thus, the price of gasoline after 1 year would be $0.65 + $0.0975 (approximately) = $0.75 per gallon. Future value interest factors can be used in calculations of this nature, and the process is exactly the same as that used to find the future value of a fixed amount. The process will also work for values less than $1. The following equation (Equation 6) can be used when the rate of price increase is known:

$$\begin{array}{l}\text{Price of a certain} \\ \text{item after n years} \\ \text{when the price is} \\ \text{increasing r per} \\ \text{year}\end{array} = \begin{array}{l}\text{Present price} \\ \text{of the item}\end{array} \times \begin{array}{l}\text{FVIF} \\ \text{(n years, r)}\end{array}$$

Equation 6

where:

FVIF (n years, r) = Entry from the Compound Sum of $1 Table, which gives the value of $1 if it is increased by r per year for n years.

EXAMPLE F: ESCALATION OF ENERGY COST.

Given: The price of natural gas in Louisiana is $2.50 per 1000 f^3 and will increase 12% per year for the next 10 years.

Find: The price of natural gas in Louisiana after the following years:

a. 1 year.
b. 6 years.
c. 10 years.

Solution:

a. Price of natural gas after 1 year when it is increasing 12% per year = Price now x FVIF (1 year, 12%)

= $2.50 x 1.1200

= $2.80.

b. Price after 6 years = 2.50 x 1.9738

= $4.93.

c. Price after 10 years = $2.50 x 3.1058

= $7.76.

Example F illustrates that cost increases are significant, especially when the price more than triples in 10 years. When the costs and cost savings of an energy project are being calculated, the possibility of rising prices must be considered. It also should be point out that energy prices are not the only costs subject to increase. In fact, most prices in the economy are increasing, but at a rate less than that of energy prices. The future price of anything can be determined with the preceding techniques, if the rate of price increase is known. With regard to energy, information concerning the rate of price increase sometimes can be obtained from utility companies and other energy producers.

CONCLUSION

Example E and the total concept of the time value of money point to the fact that energy costs should always be compared with dollar amounts of the same time period. In most energy projects, the time period used is not some year in the future, but the present. The next section of this module deals with converting dollar amounts to be received in the future to equivalent amounts associated with the present.

PRESENT VALUE OF A FIXED AMOUNT

Any amount of money to be received at some point in the future is equal to the compound sum of another smaller amount associated with the present. This smaller amount in the present is called the present value (V) of that future amount.

A good way to visualize present value is to view it as the "reverse" of the future value process.

In an earlier case, the future value of \$50, drawing interest at a rate of 8% for 3 years, was calculated to be \$62.98. (That is, \$50 in the present will become \$62.98 in 3 years if it can accumulate interest at a rate of 8%.) The present value of a future amount can be determined by taking the amount to be received at some point in the future and determining the exact amount in the present that will accumulate to that future amount, given an interest rate and time period. Therefore, in the preceding case, the present value of \$62.98 (which would be accumulated in 3 years if the interest rate is 8%) is \$50. When a future amount is reduced in value to an equivalent amount associated with some earlier time period, it is said that the amount has been discounted, and the interest rate used is called the discount rate.

DERIVATION OF A FORMULA FOR PRESENT VALUE

A formula for calculating present value can be derived by manipulating the formula already obtained for the future value of a present amount (\$P), which draws interest at rate r for n periods. The following formula for future value of a present amount was given previously as Equation 4:

$$FV_n(\$P) = \$P \times (1 + r)^n$$

Algebraically, it is possible to divide both sides of an equation without destroying the equality. In this instance, dividing both sides of the equation by $(1 + r)^n$ yields the following:

$$\frac{FV_n(\$P)}{(1 + r)^n} = \$P \qquad \text{Equation 7}$$

Note that on the right side of Equation 4, $(1 + r)^n$ is divided by $(1 + r)^n$, and the terms cancel one another. Another more convenient way to write this equation is as follows:

$$PV_n(\$F) = \frac{\$F}{(1 + r)^n} \qquad \text{Equation 8}$$

where:

$PV_n(\$F)$ = Present value of some future amount \$F to be received in n time periods.
r = Interest rate.
n = Number of time periods.

The reason Equation 7 was rewritten is because the amount being dealt with in present value calculations is associated with the future (\$F) rather than the present (\$P). With Equation 8, the present amount needed to result in the particular future value can be calculated if the interest rate and time period are known; this figure is the present value of the future amount.

EXAMPLE G: A PRESENT VALUE OF $1.

Given: One dollar is to be received in 4 years. The interest rate is 4%.

Find: The present value of this amount.

Solution:

$$PV_n(\$F) = \frac{\$F}{(1 + r)^n} \qquad \text{(Equation 8)}$$

$$FV_n(\$1) = \frac{\$1}{(1 + 0.04)^4}$$

$$= \frac{\$1}{\$1.17}$$

$$PV_n(\$1) = 85 \text{ cents.}$$

Therefore, if 85 cents is deposited in a bank account and receives 4% interest per year, there will be $1 in the account after 4 years.

HOW TO READ THE PRESENT VALUE OF $1 TABLE

As in the case of the compound sum (Data Table 1), there is also a table from which the present value of $1 can be read for various time periods and interest rates, as shown in Data Table 2. The procedure for using this table is the same as the procedure used with the future value table:

- Find the appropriate interest-rate column.
- Find the appropriate time period row.
- Find the intersection of the row and column and read the entry.

This number is the present value of $1 n years in the future, discounted at the given interest rate. A section of the Present Value of $1 Table is illustrated in Table 2.

TABLE 2. PRESENT VALUE OF $1.

Number of Periods (Time)	Rate of Interest			
	2%	3%	4%	5%
2	0.9612	0.9426	0.9246	0.9070
3	0.0423	0.9151	0.8890	0.9638
4	0.9238	0.8885	0.8548	0.8227
5	0.9057	0.8626	0.8219	0.7385

For Example G, the appropriate column to use is the one under 4%, since the relevant interest rate in that example is 4%. The appropriate row to use is the one labeled "4," since the relevant time period is 4 years. The entry where this row and this column intersect is 0.8548. Therefore, the present value of $1, to be received 4 years from now when the interest rate is 4%, is approximately 85 cents.

PRESENT VALUE OF AMOUNTS GREATER THAN $1

Table 2 can also be used to calculate the present value of future amounts greater than $1. Notice that for a given amount greater than $1, each dollar of that amount has a present value equal to the amount found in the Present Value of $1 Table. Therefore, to utilize Table 2 for amounts greater than $1, use the following procedure:

- Obtain the time period and relevant interest rate.
- In the table locate the present value of $1 for this time period and interest rate. This is the PVIF.

- Multiply this PVIF by the original future amount. The product is the present value of that amount.

Since the entries from the Present Value of $1 Table can be used to multiply by a future amount to get its present value, these entries sometimes are called Present Value Interest Factors (PVIF). A particular factor associated with an interest rate r and n year is designated by PVIF (n years, r). Therefore, another equation for the present value of a future amount $FV_n(\$M)$ is given as follows:

$$PV_n(\$F) = \$F \times PVIF\ (n\ years,\ r) \qquad \text{Equation 9}$$

where:

$PV_n(\$F)$ = Present value of a future amount $F to be received in n years from the present.

$F = Some future amount associated with a time period n years from the present.

PVIF (n years, r) = Present value of $1 to be received in n years when the interest rate is r – also called the Present Value Interest Factor.

This latter method (Equation 9) is much easier to use than that described by Equation 7.

EXAMPLE H: PRESENT VALUE OF AN AMOUNT GREATER THAN $1.

Given: A newly installed compressor is to be replaced in 5 years. At that time, it is estimated that the compressor can be sold for $200. The interest rate is 4%.

Find: The present value of the selling price.

Solution: From Equation 8, recall the following:

$$PV_n(\$F) = \$F \times PVIF\ (5\text{ years, }3\%)$$

$$\begin{aligned} PV_n(\$200) &= \$200 \times 0.8262 \\ &= \$172.52. \end{aligned}$$

Thus, $200, to be received 5 years from now, is worth only $172.52 today.

Previously, Example E showed a business that had the choice of paying either $2000 now or $3000 in 5 years for two solar collector panels — with a relevant interest rate of 10%. The following calculations show how these amounts can be converted to present value and compared: The present value of $2000 paid now is just $2000. The present value of $3000 5 years from now, discounted at 10%, is equal to $(3000)/(1.10)^5 = (\$3000) \times (0.6209) = \$1862.70$. The figure (0.6209), in this case, is just the present value of $1 to be received in 5 years, discounted back at 10%. (The figure 0.6209 was obtained from Data Table 2.) The implication of these calculations is that paying $3000 in 5 years, when the relevant interest rate is 10%, is exactly the same as paying $1862.70 now. Given the choice of paying $2000 now or $1862.70 now, the obvious choice would be to pay the $1862.70.

This implies that the alternative of paying $3000 in 5 years should be chosen over the alternative of paying $2000 now, since $1862.70 could be deposited in the bank now and, in effect, would result in $3000 in 5 years. Therefore, the savings are equal to $2000 - $1862.70 = $137.30 in present value terms.

It may be recalled that in the first presentation of Example E, the savings associated with choosing the $3000 plan were equal to $221, to be received in 5 years. The present value of this amount, when the interest rate is 10%, is equal to ($221) x (0.6209) = $137.22 — which approximately is equal to the savings obtained by the other method. (The slight difference between the two amounts is due to rounding of numbers.)

The important implication of this example is that the present value calculation and the compound sum calculation led to the same dollar amount, and, in both cases, the dollar amounts were converted to comparable time periods.

OBSERVATIONS CONCERNING THE PRESENT VALUE OF A FIXED AMOUNT

Careful examination of Data Table 2 (a more extensive Present Value of $1 Table located near the end of this module) reveals two facts:

- If an amount is discounted back to a higher rate of interest, its present value is smaller.
- The present value of an amount to be received in the future is smaller the farther in the future it is to be received.

The first fact is explained as follows: The present value (original amount) is smaller at higher interest rates — the reason being that each year the higher interest rate results in a larger percentage of the amount being added to the sum. The second fact is applicable because a longer time period gives the original amount more time to accumulate. The more time the original amount has to accumulate, the smaller the amount can be when it begins to accumulate. The relationships among present value, rate of interest, and time are illustrated in Figure 6.

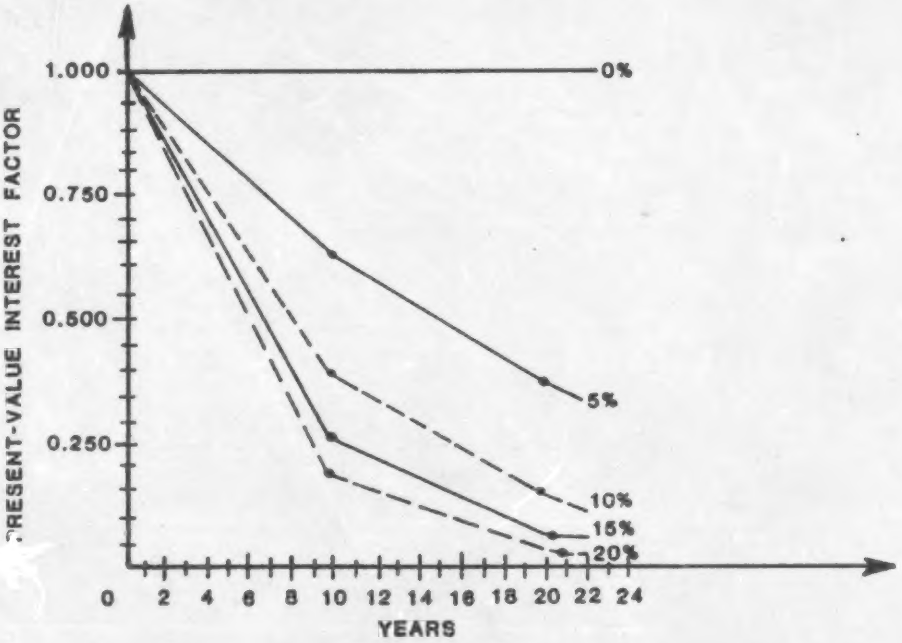

Figure 6. Present Value of $1.

SHORTER TIME PERIODS

Occasionally, a situation will arise where a time period shorter than 1 year must be used. The following time periods are examples:

1 month = 1/12 year
1 week = 1/52 year
1 day = 1/360 year

Situations where these time periods are used are easy to compute with the Compound Sum of $1 Table (and, thus, the Present Value of $1 Table). These situations apply to time periods, not just years. The important thing to remember when using this table is that the interest rate must always correspond to the time period. For instance, an interest rate of 8% per year is not the same as an interest rate of 8% per month. The easiest way to convert an interest rate expressed in years to a shorter time period is to use the figures given previously. For instance, an interest rate of 8% per year is the same as an interest rate of 8/12 = 0.67 per month. (The appropriate number of time periods are used — for example, one year is 12 months, and so forth.) The following example (Example I) shows how this technique can be useful:

EXAMPLE I: SHORTER TIME PERIODS AND COST ESCALATION.

Given: A homeowner's electricity bill in January is \$35. The price of electricity is rising at the rate of 12% per year.

Find: The homeowner's bill in June (5 months later), assuming the use of the same amount of electricity and that rates increase monthly.

Solution: An interest rate of 12/12 = 1% per month; therefore, the bill will increase at the rate of 1% per time period for five time periods. Recall Equation 6, and use the word "months" instead of "years," as follows:

Electricity bill after 5 months when it is increasing 1% per month = Present bill x FVIF (5 months, 1% per month)

= \$35 x 1.0510

= \$36.79.

The techniques presented in this module provide the fundamental skills necessary for accurate analysis of the costs and cost savings of energy conservation projects. These techniques must be firmly understood by the energy specialist.

EXERCISES

1. A steam-tracing line has a one-eighth inch diameter hole from which steam is escaping. The plant uses 540 million Btus per year to heat the steam lost through this hole.
    a. At $1.20 per million Btus (MBtu), what is the cost of the leak to the plant?
    b. If the inflation rate is 6%, what would be the cost to the plant next year if the leak is unrepaired?
    c. If the inflation rate is 6%, what would be the cost to the plant 4 years from now if the leak is unrepaired? What is the present value (PV) of that cost?
    d. Complete the chart in Figure 7.
    e. Use the chart in Figure 7 to answer the following questions:
        (1) Which alternative, A, B, or C, has the greatest total losses?
        (2) Which alternative has the greatest sum of PV of losses?
        (3) What factors cause the sum of PV of losses to differ from total losses?

YEAR	A		B		C	
	ANNUAL LOSS @ $1.20/MBtu	PV yearly 6% = r	ANNUAL LOSS x FVIF Begin with $1.20/MBtu and escalated 4%/year	x PVIF = PV yearly r = 6%	ANNUAL LOSS x FVIF Begin with $1.20/MBtu and escalated 8%/year	x PVIF = PV yearly r = 6%
1						
2						
3						
4						
5						
6						
Total Losses						
Sum of PV of Losses						

Figure 7. Annual Loss Chart.

DATA TABLE 1: FUTURE VALUE OF $1 (FVIF).

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1200	1.1400	1.1500	1.1600	1.1800	1.2000	1.2400	1.2800	1.3200	1.3600
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2544	1.2996	1.3225	1.3456	1.3924	1.4400	1.5376	1.6384	1.7424	1.8496
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.4049	1.4815	1.5209	1.5609	1.6430	1.7280	1.9066	2.0972	2.3000	2.5155
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5735	1.6890	1.7490	1.8106	1.9388	2.0736	2.3642	2.6844	3.0360	3.4210
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.7623	1.9254	2.0114	2.1003	2.2878	2.4883	2.9316	3.4360	4.0075	4.6526
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.9738	2.1950	2.3131	2.4364	2.6996	2.9860	3.6352	4.3980	5.2899	6.3275
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.2107	2.5023	2.6600	2.8262	3.1855	3.5832	4.5077	5.6295	6.9826	8.6054
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.4760	2.8526	3.0590	3.2784	3.7589	4.2998	5.5895	7.2058	9.2170	11.703
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.7731	3.2519	3.5179	3.8030	4.4355	5.1598	6.9310	9.2234	12.166	15.916
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	3.1058	3.7072	4.0456	4.4114	5.2338	6.1917	8.5944	11.806	16.059	21.646
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.4785	4.2262	4.6524	5.1173	6.1759	7.4301	10.657	15.111	21.198	29.439
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.8960	4.8179	5.3502	5.9360	7.2876	8.9161	13.214	19.342	27.982	40.037
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	4.3635	5.4924	6.1528	6.8858	8.5994	10.699	16.386	24.758	36.937	54.451
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.8871	6.2613	7.0757	7.9875	10.147	12.839	20.319	31.691	48.756	74.053
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	5.4736	7.1379	8.1371	9.2655	11.973	15.407	25.196	40.564	64.358	100.71
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	6.1304	8.1372	9.3576	10.748	14.129	18.488	31.242	51.923	84.953	136.96
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	6.8660	9.2765	10.761	12.467	16.672	22.186	38.740	66.461	112.13	186.27
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	7.6900	10.575	12.375	14.462	19.673	26.623	48.038	85.070	148.02	253.33
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	8.6128	12.055	14.231	16.776	23.214	31.948	59.567	108.89	195.39	344.53
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	9.6463	13.743	16.366	19.460	27.393	38.337	73.864	139.37	257.91	468.57
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	10.803	15.667	18.821	22.574	32.323	46.005	91.591	178.40	340.44	637.26
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	12.100	17.861	21.644	26.186	38.142	55.206	113.57	228.35	449.39	866.67
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	13.552	20.361	24.891	30.376	45.007	66.247	140.83	292.30	593.19	1178.6
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	15.178	23.212	28.625	35.236	53.108	79.496	174.63	374.14	783.02	1602.9
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.834	17.000	26.461	32.918	40.874	62.668	95.396	216.54	478.90	1033.5	2180.0
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.918	19.040	30.166	37.856	47.414	73.948	114.47	268.51	612.99	1364.3	2964.9
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.245	13.110	21.324	34.389	43.535	55.000	87.259	137.37	332.95	784.63	1800.9	4032.2
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.167	14.421	23.883	39.204	50.065	63.800	102.96	164.84	412.86	1004.3	2377.2	5483.8
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.172	15.863	26.749	44.693	57.575	74.008	121.50	197.81	511.95	1285.5	3137.9	7458.0
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.062	13.267	17.449	29.959	50.950	66.211	85.849	143.37	237.37	634.81	1645.5	4142.0	10143.
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.285	14.974	21.724	31.409	45.259	93.050	188.88	267.86	378.72	750.37	1469.7	5455.9	19426.	66520.	*
50	1.6446	2.6916	4.3839	7.1067	11.467	18.420	29.457	46.901	74.357	117.39	289.00	700.23	1083.6	1670.7	3927.3	9100.4	46890.	*	*	*
60	1.8167	3.2810	5.8916	10.519	18.679	32.987	57.946	101.25	176.03	304.48	897.59	2595.9	4383.9	7370.1	20555.	56347.	*	*	*	*

*FVIF > 99,999.

DATA TABLE 2: PRESENT VALUE OF $1 (PVIF).

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174	.9091	.8929	.8772	.8696	.8621	.8475	.8333	.8065	.7813	.7576	.7353
2	.9803	.9612	.9426	.9246	.9070	.8900	.8734	.8573	.8417	.8264	.7972	.7695	.7561	.7432	.7182	.6944	.6504	.6104	.5739	.5407
3	.9706	.9423	.9151	.8890	.8638	.8396	.8163	.7938	.7722	.7513	.7118	.6750	.6575	.6407	.6086	.5787	.5245	.4768	.4348	.3975
4	.9610	.9238	.8885	.8548	.8227	.7921	.7629	.7350	.7084	.6830	.6355	.5921	.5718	.5523	.5158	.4823	.4230	.3725	.3294	.2923
5	.9515	.9057	.8626	.8219	.7835	.7473	.7130	.6806	.6499	.6209	.5674	.5194	.4972	.4761	.4371	.4019	.3411	.2910	.2495	.2149
6	.9420	.8880	.8375	.7903	.7462	.7050	.6663	.6302	.5963	.5645	.5066	.4556	.4323	.4104	.3704	.3349	.2751	.2274	.1890	.1580
7	.9327	.8706	.8131	.7599	.7107	.6651	.6227	.5835	.5470	.5132	.4523	.3996	.3759	.3538	.3139	.2791	.2218	.1776	.1432	.1162
8	.9235	.8535	.7894	.7307	.6768	.6274	.5820	.5403	.5019	.4665	.4039	.3506	.3269	.3050	.2660	.2326	.1789	.1388	.1085	.0854
9	.9143	.8368	.7664	.7026	.6446	.5919	.5439	.5002	.4604	.4241	.3606	.3075	.2843	.2630	.2255	.1938	.1443	.1084	.0822	.0628
10	.9053	.8203	.7441	.6756	.6139	.5584	.5083	.4632	.4224	.3855	.3220	.2697	.2472	.2267	.1911	.1615	.1164	.0847	.0623	.0462
11	.8963	.8043	.7224	.6496	.5847	.5268	.4751	.4289	.3875	.3506	.2875	.2366	.2149	.1954	.1619	.1346	.0938	.0662	.0472	.0340
12	.8874	.7885	.7014	.6246	.5568	.4970	.4440	.3971	.3555	.3186	.2567	.2076	.1869	.1685	.1372	.1122	.0757	.0517	.0357	.0250
13	.8787	.7730	.6810	.6006	.5303	.4688	.4150	.3677	.3262	.2897	.2292	.1821	.1625	.1452	.1163	.0935	.0610	.0404	.0271	.0184
14	.8700	.7579	.6611	.5775	.5051	.4423	.3878	.3405	.2992	.2633	.2046	.1597	.1413	.1252	.0985	.0779	.0492	.0316	.0205	.0135
15	.8613	.7430	.6419	.5553	.4810	.4173	.3624	.3152	.2745	.2394	.1827	.1401	.1229	.1079	.0835	.0649	.0397	.0247	.0155	.0099
16	.8528	.7284	.6232	.5339	.4581	.3936	.3387	.2919	.2519	.2176	.1631	.1229	.1069	.0930	.0708	.0541	.0320	.0193	.0118	.0073
17	.8444	.7142	.6050	.5134	.4363	.3714	.3166	.2703	.2311	.1978	.1456	.1078	.0929	.0802	.0600	.0451	.0258	.0150	.0089	.0054
18	.8360	.7002	.5874	.4936	.4155	.3503	.2959	.2502	.2120	.1799	.1300	.0946	.0808	.0691	.0508	.0376	.0208	.0118	.0068	.0039
19	.8277	.6864	.5703	.4746	.3957	.3305	.2765	.2317	.1945	.1635	.1161	.0829	.0703	.0596	.0431	.0313	.0168	.0092	.0051	.0029
20	.8195	.6730	.5537	.4564	.3769	.3118	.2584	.2145	.1784	.1486	.1037	.0728	.0611	.0514	.0365	.0261	.0135	.0072	.0039	.0021
25	.7798	.6095	.4776	.3751	.2953	.2330	.1842	.1460	.1160	.0923	.0588	.0378	.0304	.0245	.0160	.0105	.0046	.0021	.0010	.0006
30	.7419	.5521	.4120	.3083	.2314	.1741	.1314	.0994	.0754	.0573	.0334	.0196	.0151	.0116	.0070	.0042	.0016	.0006	.0002	.0001
40	.6717	.4529	.3066	.2083	.1420	.0972	.0668	.0460	.0318	.0221	.0107	.0053	.0037	.0026	.0013	.0007	.0002	.0001	*	*
50	.6080	.3715	.2281	.1407	.0872	.0543	.0339	.0213	.0134	.0085	.0035	.0014	.0009	.0006	.0003	.0001	*	*	*	*
60	.5504	.3048	.1697	.0951	.0535	.0303	.0173	.0099	.0057	.0033	.0011	.0004	.0002	.0001	*	*	*	*	*	*

*The factor is zero to four decimal places

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GLOSSARY

Compounding: The process in which the interest of an amount of money earns interest itself in subsequent years.

Cost escalation: The calculation of the impact of inflation on the future price of resources.

Discount: The process of calculating the present value of future cash flows at a specified interest rate (the "discount rate").

Discount rate: The interest rate used to calculate the present value of future income.

Future value: The dollar figure an amount of money will become if invested at a specified interest rate for a specified time period.

Future value interest factor: A number, when multiplied by an initial amount that will yield the amount the principal would grow to if invested at a specified interest rate. It is equal to $(1 + r)^t$.

Interest: Dollar amount borrower pays lender for the use of lender's money.

Interest rate: Figure indicating the portion of the principal that must be paid in interest over each time period.

Present value: The value now of a sum to be paid or received at a specified future data.

Present value interest factor: A number that, when multiplied by a future amount, will yield the initial amount of money that would grow to the future amount if invested at a specified interest rate. It equals $\frac{1}{(1 + r)^t}$.

Principal: The amount of money borrowed to replace, install, or modify an energy system.

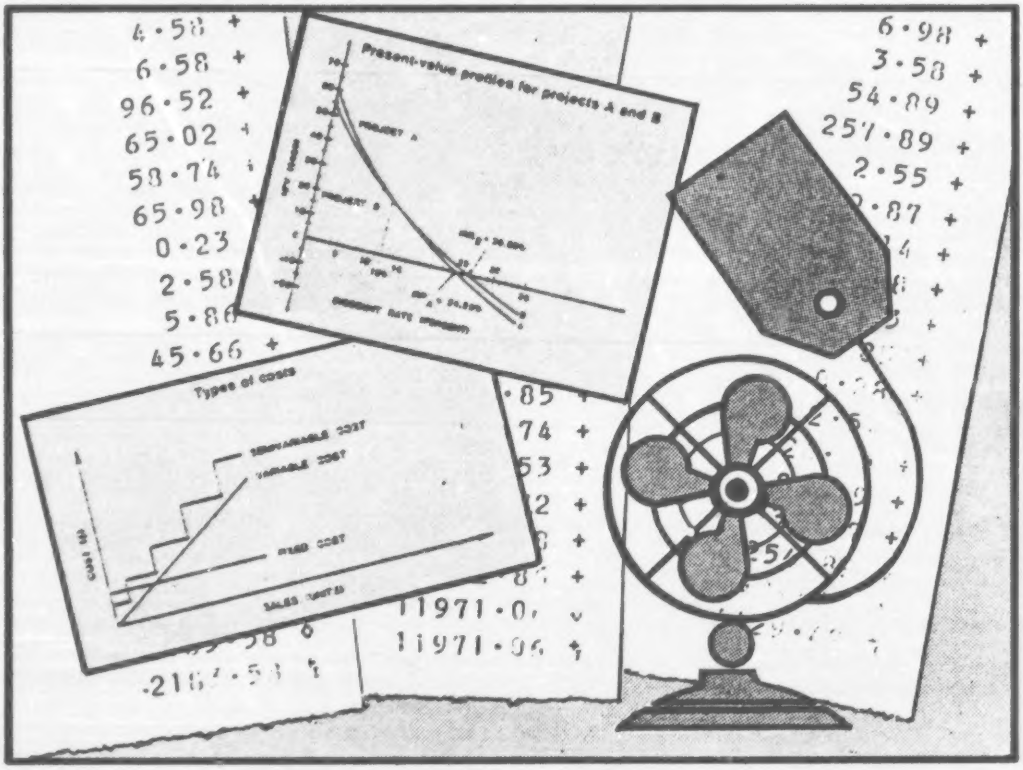
Time period: The length of time, usually a year, associated with interest rates.

Time value of money: The concept that a dollar is worth more today than in the future because a dollar received today can be invested to yield more than a dollar in the future.

ENERGY TECHNOLOGY

CONSERVATION AND USE

ENERGY ECONOMICS



EE-03

FINANCIAL TECHNIQUES OF ENERGY ECONOMICS

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INTRODUCTION

This module discusses the techniques that can be used in the analysis of cost and cost savings of energy projects and is an extension of the techniques presented in previous modules. In Module EE-01, "Fundamentals of Energy Cost Analysis," a distinction was made between one-time costs and recurring costs. It was learned that one-time costs involve one simple, direct payment, whereas recurring costs can involve a number of payments over a period of time. Module EE-02, "Financial Parameters of Energy Economics," introduced techniques which can be applied to one-time costs.

The techniques presented in this module primarily apply to recurring costs and cost savings and to different situations with regard to when and how often costs and cost savings occur. With the skills that can be learned from these techniques, the energy specialist will be able to compute the present value of costs and cost savings, an important consideration in energy projects. The relevance of the concept of present value is emphasized throughout the module.

PREREQUISITES

The student should have a good understanding of basic algebraic functions and should have completed Modules EE-01 and EE-02 of Energy Economics.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Define the following terms:
    a. Annuity.
    b. Irregular flow of costs and cost savings.
2. Given the proper information, compute the following:
    a. The sum of an annuity of any amount.
    b. The present value of an annuity of any amount.
    c. The present value of an unending annuity of any amount.
    d. The present value of an irregular flow of cost savings.
3. Extend the concept of present value to include cost escalation.
4. Explain the importance of present value in the evaluation of the cost and cost savings of energy projects.

COST OF AN ANNUITY

Most projects that the energy specialist will encounter involve a series of cost savings, as opposed to a one-time amount as per the present value equation. For instance, a microcomputer system that automatically adjusts the temperatures of unoccupied areas of a building in order to conserve energy – and thereby reduce heating and cooling costs – will not generate one-time savings but, rather, an annual stream of savings over the life of the system. This flow of dollars must be standardized, because savings will occur at different times. If the system saves $4000 per year, then the $4000 saved the first year will be more valuable than the $4000 saved the second year; the $4000 saved the second year will be more valuable than the $4000 saved the third year; and so on. The present value of this flow of savings must be calculated so that it can be compared to the cost of the microcomputer system.

Analysis of this nature makes use of the concept of an annuity. An annuity is simply a pattern of cash flow that is equal in each year – every year the same dollar amount is received or spent. First, the way in which an annuity accumulates, or sums, to a future value will be considered; then, this analysis will be extended to develop a formula for the calculation of the present value of an annuity.

ACCUMULATION OF AN ANNUITY

Consider a company debating the purchase of a microcomputer system. This system has an estimated life of 4 years, with cost savings of $4000 per year. Thus, the savings form a 4-year annuity of $4000. Suppose that these savings are deposited in a bank account drawing an 8% rate of interest per year, and that the total dollar value of the account at the end of 4 years is to be calculated. The procedure for the accumulation of this annuity is as follows:

- At the end of the first year, $4000 in cost savings is deposited in the account. During the next year, this earns $4000 x 0.08 = $320. Thus, the account has accumulated $4320.
- At the end of the second year, the cost savings from the second year are added to the account to bring the total to $8320. This amount earns $8320 x 0.08 = $665.60 in interest the third year – which brings the amount in the account to $8985.60.
- At the end of the third year, the cost savings from the third year are added to the account to bring the total to $12,985.60. This amount earns $12,985.60 x 0.08 = $1038.85 interest during the fourth year. This brings the total to $14,024.45.
- At the end of the fourth year, the savings from the fourth year are added to bring the total to $18,024.45. Since 4 years have elapsed, this marks the end of the annuity. This figure (total) is the future value of the cost savings associated with the installation of the microcomputer system.

Another way to view the compounding of this annuity is to examine the number of years each amount of cost savings can earn interest. The cost savings of the first year can draw interest for 3 years; those of the second year can draw interest for 2 years; and so on. The sum of the annuity is found by obtaining the compound sum of each payment. This process is shown in Table 1.

TABLE 1. SUM OF AN ANNUITY ($4000, 4 YEARS, 8%).

End of Year	Amount Deposited	Number of Years Compounded	Appropriate Future Value Interest Factor (FVIF)*	Future Value
1	$4000	3	1.2597	$5038.80
2	$4000	2	1.1664	4665.60
3	$4000	1	1.0800	4320.00
4	$4000	0	1.000	4000.00
Amount at the End of Year 4 = $18024.40				

*The FVIF figures were obtained from Data Table 1.

SUM OF AN ANNUITY OF $1 FORMULA

The special case of an annuity of $1 will be considered in order to generalize the preceding process into a formula. This is the receipt of $1 per year for a given number of years (designated as n) with an applicable interest rate r. At the end of the first year, the dollar deposited will earn interest for (n-1) years and, at the end of the second year, will earn interest for (n-2) years. This pattern continues for every year up to the dollar deposited in the (n-1) year (next-to-last year) — which will earn interest for 1 year. The dollar deposited at the end of the last year will earn no interest since the time period is past. This process is shown in Table 2.

TABLE 2. NUMBER OF YEARS INTEREST EARNED BY EACH PAYMENT IN AN ANNUITY.

Deposited End of Year	Amount Deposited in Account	Years Interest Earned
1	$1	n-1
2	$1	n-2
3	$1	n-3
4	$1	n-4
.	.	.
.	.	.
.	.	.
n-1	$1	1
n	$1	0

Note: Periods indicate possible, sequential missing numbers.

The total amount in the bank account at the end of n years can be calculated by determining the future of these amounts as follows:

FVA_n (\$1) = Future value of \$1 n-1 periods later
+ Future value of \$1 n-2 periods later
+ ...*
+ Future value of \$1 one period later
+ \$1.

*Note: The ellipsis above indicates missing numbers.

Whenever ... (the ellipsis) is used, it simply signifies that a series of numbers or words is not being written. For instance, instead of writing 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, it is much easier to write 1, 2, ..., 10 – with the understanding that all of the missing numbers are included. In the left-hand column of Table 2, the ... (in a vertical line) signifies all the numbers between 4 and n-1, whatever n-1 might be. If n is 50, then n-1 is 49, and the ⋮ signifies the numbers 5 through 49. In many cases, such as the one above, this method is more convenient. Similarly, if the n (in Table 2) is 7, then n-1 is 6, and the ellipsis (...) simply signifies the number 5. The use of this technique is made possible because n is a variable that can take on many values.

Therefore, the following may be written:

$$FVA_n(\$1) = (1 + r)^{n-1} + (1 + r)^{n-2} + \dots + (1 + r)^1 + \$1 \qquad \text{Equation 1}$$

where:

FVA_n(\$1) = Sum (or future value) of an annuity of \$1 after it has been paid for n time periods.
r = Interest rate.

EXAMPLE A: FUTURE VALUE OF AN ANNUITY OF $1.

Given: One dollar is deposited in a bank account at the end of the year for 4 consecutive years. The interest rate is 4%.

Find: The future value of the annuity after 4 years.

Solution: Use Equation 1 as follows:

$$\begin{aligned} FVA_4(\$1) &= (1 + r)^3 + (1 + r)^2 + (1 + r)^1 + 1 \\ &= (1 + 0.04)^3 + (1 + 0.04)^2 \\ &\quad + (1 + 0.04)^1 + 1 \\ &= (1.04)^3 + (1.04)^2 + (1.04)^1 + 1 \\ &= 1.125 + 1.081 + 1.04 + 1 \\ &= \$4.246 \\ FVA_4(\$1) &= \$4.25. \end{aligned}$$

THE SUM (FUTURE VALUE) OF AN ANNUITY OF $1 TABLE

Admittedly, performing calculations of this nature could become very tedious. Fortunately, a table exists for finding the sum (future value) of an annuity of $1 for different interest rates and time periods (see Table 3 for an example). The steps in the use of Table 3 are as follows:

- Find the column that corresponds to the appropriate interest rate.
- Find the row that corresponds to the number of time periods in which payments will be made.
- Find the intersection of the row and the column and read the entry. This is the sum of an annuity of $1 for the appropriate number of periods and interest rate.

TABLE 3. SUM OF AN ANNUITY OF $1 PER TIME PERIOD.

Number of Periods (Time)	Rate of Interest			
	1%	2%	3%	4%
1	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400
3	3.0301	3.0604	3.0909	3.1216
4	4.0604	4.1216	4.1836	4.2465

For example, the sum of an annuity of $1 per year for 4 years – when the interest rate is 4% – is obtained by finding the intersection of the 4% column and the time row labeled "4." This entry is $4.2465, which is the value sought. A more extensive table that can be used to find the future value of an annuity of $1 is located in the back of this module (Data Table 3).

SUM OF AN ANNUITY OF MORE THAN $1 PER YEAR

What if the annuity is more than $1 per year? Each dollar in the annuity will have the same value as the sum of an annuity of $1 after the given number of time periods (which accumulates at the given interest rate). Therefore, all that is required is to find the sum of the appropriate $1 annuity, followed by the multiplication of this number by the original dollar value of the annuity. The result is the sum of the annuity of the given amount. For this reason, the appropriate present value of an annuity of $1 figure is often called the Future Value Interest Factor of an Annuity. This term is abbreviated $FVIF_a$, where the subscript "a" indicates that an annuity – rather than a single amount – is involved. The equation for finding the future value of an annuity greater than $1 is as follows:

$$FVA_n(\$M) = \$M \times FVIF_a(n \text{ years}, r) \qquad \text{Equation 2}$$

where:

$FVA_n(\$M)$ = Future value (sum) of an annuity of $M, which accumulates for n years at r.

$M = Amount of the annuity.

$FVIF_a(n \text{ years}, r)$ = Future value of an annuity of $1, which accumulates for n years at r. This is also called the Future Value Interest Factor of an Annuity. (This can be obtained from Data Table 3).

n = Number of years the annuity is received.

r = Interest rate.

The process of finding the appropriate $FVIF_a$ to compute the sum of an annuity of more than $1 is demonstrated in Example B.

EXAMPLE B: SUM OF AN ANNUITY OF MORE THAN $1 PER YEAR.

Given: Each of the elevators in a building is equipped with 300 watts (W) of fluorescent lighting, although only 30 W are needed. The lights burn 24 hours per day (24 h/d), 365 days per year (d/yr). The cost of electricity is $0.03 per kilowatt hour (kWh), and the interest rate is 8%.

Find: The cost savings per year resulting from the reduction in watts in one elevator and the future value of these cost savings after 4 years.

Solution: The lights burn 24 h/d and 365 d/hr; therefore,

$$24\ \frac{h}{d} \times 365\ \frac{h}{yr} = 8760\ \frac{h}{yr} \text{ electricity usage}$$

$$8760\ \frac{h}{yr} \times 270 \text{ W saved} = 2{,}365{,}200\ \frac{\text{watthour (Wh)}}{yr}.$$

And, since 1 kWh = 1000 Wh, savings in electricity = 2,365.2 kWh/yr.

Since the cost of electricity is $0.03/kWh, the cost savings per year are given by the following:

$$2{,}365.2\ \frac{kWh}{yr} \times \$0.03/kWh = \$70.96 \text{ per year cost savings.}$$

Thus, the savings associated with this project form an annuity of $70.96 per year. With an 8% interest rate, the future value of this annuity after 4 years is obtained as follows (using Equation 2):

Example B. Continued.

$$FVA_n(\$70.96) = \$70.96 \times FVIF_a(4 \text{ years}, 8\%)$$
$$= \$70.96 \times 4.5061$$
$$FVA_n(\$70.96) = \$319.75.$$

The value for $FVIF_a$(4 years, 8%) was obtained from Data Table 3 (back of module).

PRESENT VALUE OF AN ANNUITY

Whereas the idea of the sum of an annuity seldom is used in energy project decisions directly, it is useful in understanding one of the most important concepts in energy economics — the idea of the present value of an annuity. As in the case of the present value of a fixed amount being the opposite of the future value of a fixed amount, so is the present value of an annuity the opposite of the future value (sum) of an annuity. The present value of an annuity is the dollar value that answers this question: What fixed amount today will accumulate after the given number of years to exactly the same amount as the sum (future value) of the annuity?

CALCULATION OF THE PRESENT VALUE OF AN ANNUITY

The present value of an annuity can be calculated in the following way: Each amount that is added to the annuity each year is discounted back to the present. These individual amounts should then be summed to obtain the total present value of the annuity. At this point, Example C will be helpful.

EXAMPLE C: THE PRESENT VALUE OF AN ANNUITY.

Given: A plan to reduce the electricity cost associated with lighting involves the replacement of bulbs of a certain wattage with bulbs of a lower wattage in every hallway of a hotel. This system has an expected life of 8 years, and this process will save $1300 per year for the 8-year period. The interest rate is 6%.

Find: The present value of the cost savings.

Solution: The cost savings form an annuity of $1300 per year for 8 years. The cost savings associated with each year will be discounted individually back to the present, and the sum will be taken.

Each present value calculation is identical to those calculated in Module EE-02. This process is shown in Table 4. (The PVIF figures were obtained from Data Table 2.)

TABLE 4. THE PRESENT VALUE OF AN 8-YEAR ANNUITY OF $1300 WITH INTEREST RATE 6%.

Year	Cost Savings	PVIF	Present Value
1	$1300	0.9434	$1226.42
2	$1300	0.8900	$1157.00
3	$1300	0.8396	$1091.48
4	$1300	0.7921	$1029.73
5	$1300	0.7473	$ 971.49
6	$1300	0.7050	$ 916.50
7	$1300	0.6651	$ 864.63
8	$1300	0.6274	$ 815.62
Present Value of Annuity = $8072.87			

PRESENT VALUE OF AN ANNUITY OF $1

Just as it was in the case of the sum of an annuity, it would be useful at this point to examine the special case of an annuity of $1. This value can be calculated in a manner similar to that used in Table 4, except that the amount in question is $1 instead of $1300. In this case, the present value of the cost savings of each year is just the appropriate entry from the Present Value of $1 Table (the appropriate PVIF). Therefore, the present value of an annuity of $1 for n years – when the interest rate is just the sum of the present values of the cost savings for each year – is as follows:

$$PVA_n(\$1) = PVIF\ (1\ year,\ r) + PVIF\ (2\ years,\ r) + \ldots\ldots + PVIF\ (n\ years,\ r) \qquad \text{Equation 3}$$

where:

$PVA_n(\$1)$ = Present value of an annuity of $1 for n time periods.

n = Number of time periods the annuity is received.

r = Interest rate.

PVIF = Present value interest factor (the present value of $1) associated with the given interest rate and year in the future.

Note that PVIF (1 year, r) is just the present value of $1 received after 1 year; the PVIF (2 years, r) is the present value of $1 received after 2 years; and so on. If the present values of the cost savings of each year are summed, the result is the present value of the annuity.

EXAMPLE D: PRESENT VALUE OF AN ANNUITY OF $1.

Given: One dollar per year is to be received at the end of the next 4 years. The interest rate is 6%.

Find: The present value of the annuity, using Data Table 2.

Solution: Use Equation 3 as follows:

$$\begin{aligned} PVA_4(\$1) &= PVIF\ (1\ year,\ 6\%) + PVIF\ (2\ years,\ 6\%) + PVIF\ (3\ years,\ 6\%) + PVIF\ (4\ years,\ 6\%) \\ &= 0.9434 + 0.8900 + 0.8396 + 0.7921 \\ PVA_4(\$1) &= \$3.4651. \end{aligned}$$

THE PRESENT VALUE OF AN ANNUITY OF $1 TABLE

The Present Value of an Annuity of $1 Table (Data Table 4) is simply a collection of the preceding sums for many possible time periods and interest rates. The procedure for reading Data Table 4 is as follows:

- Find the column corresponding to the appropriate interest rate.
- Find the row corresponding to the appropriate number of years.
- Find the point where the row and column intersect and read the entry. This is the present value figure.

PRESENT VALUE OF ANNUITIES LARGER THAN $1

In an annuity larger than $1, each dollar will accumulate exactly like the annuity of $1. Therefore, the method of obtaining the present value of an annuity of more than $1 is as follows:

- Find the present value of an annuity of $1 that covers the same number of time periods and has the same interest rate.
- Multiply this number by the amount of the larger annuity.

Because this procedure can be used, the entry from the Present Value of an Annuity of $1 Table is often called the Present Value Interest Factor of an Annuity. This term is abbreviated $PVIF_a$, where the subscript "a" means the number applies to an annuity rather than a fixed amount. The following equation can be used to find the present value of an annuity of an amount $M:

$$PVA_n(\$M) = \$M \times PVIF_a(n \text{ years}, r) \qquad \text{Equation 4}$$

where:

$PVA_n(\$M)$ = Present value of an annuity of $M that is received for n time periods.

$M = Amount of the annuity (of each payment).

n = Number of time periods (years) the annuity accumulates.

r = Interest rate.

PVIF_a(n year, r) = Appropriate present value interest factor (obtained from Data Table 4), or the present value of an annuity of \$1 that accumulates for n time periods when the interest rate is r per time period.

Example E demonstrates how Equation 4 is used.

EXAMPLE E: CALCULATION OF THE PRESENT VALUE OF AN ANNUITY.

Given: Energy conservation measures which will result in savings of \$700 per year for 5 years. The interest rate is 9%.

Find: The present value of the annuity formed by these cost savings.

Solution: First, examine the Present Value of an Annuity of \$1 Table. (Part of Data Table 4 is reproduced here for convenience as Table 5.)

TABLE 5. THE PRESENT VALUE OF AN ANNUITY OF \$1.

Number of Payment	Rate of Interest			
	7%	8%	9%	10%
4	3.3872	3.3121	3.2397	3.1699
5	4.1002	3.9927	3.8897	3.7908
6	4.7665	4.6229	4.4859	4.3553
7	5.3893	5.2064	5.0330	4.8684

Example E. Continued.

In this case, the appropriate column is 9% and the appropriate row is for 5 years (time periods). The entry where the row and column intersect is 3.8897; therefore, the present value of the annuity, formed by the cost savings associated with the conservation measure, can be calculated by using Equation 4 as follows:

$$PVA_5(\$700) = \$700 \times PVIF_a \text{ (5 years, 9\%)}$$
$$= \$700 \times 3.8897$$
$$PVA_5(\$700) = \$2722.79.$$

(The $PVIF_a$ figure was obtained from Data Table 4.)

ANALYSIS OF ENERGY PROJECTS, USING PRESENT VALUE OF AN ANNUITY

In general, the energy specialist should employ these concepts of energy economics when any course of action is under consideration which involves monetary costs and/or costs savings. Once the costs and cost savings (benefits) have been determined, they can be converted to present value and compared with one another to determine if the project is worthwhile – that is, if the cost savings of the project are greater than its costs in present value terms. The following example (Example F) is a detailed example of how the preceding techniques can be employed in the analysis of an energy project.

EXAMPLE F: PRESENT VALUE OF AN ANNUITY OF COST SAVINGS.

Given: A building occupies 5100 square feet (5.1×10^3 ft^2) of floor space, and the number of degree-days/yr is 2363. A 10°F reduction in the temperature of the building at night during the colder part of the year would save 20,000 Btus per square feet (2×10^4 Btu/ft^2) of floor space per year. The heating system is fueled by natural gas, and each cubic foot (ft^3) of natural gas has 1000 Btus of heat energy. The price of natural gas is $2.57 per thousand cubic feet (\$2.57/M ft^3). The heating unit will be operational for an additional 20 years, and the interest rate is 10%.

Find: The cost savings per year associated with this conservation measure, and the present value of the annuity of cost savings.

Solution: 2×10^4 $Btu/ft^2/yr$ $\times$ 5100 ft^2 = 10.2×10^7 Btu/yr total heat savings.

Since each cubic foot of natural gas has 1000 Btus of heat content,

10.2×10^7 Btu/yr $\div$ 1000 Btu/ft^3 = 10.2×10^4 ft^3/yr natural gas savings.

Natural gas cost is \$2.57/M ft^3 = \$0.00257/ft^3,

10.2×10^4 ft^3/yr $\times$ \$0.00257/ft^3 = \$262.11 savings per year.

These yearly cost savings form a 20-year annuity of \$262.11 per year.

Therefore, Equation 4 can be used to calculate the present value as follows:

Example F. Continued.

$$PVA_{20}(\$262.11) = \$262.11 \times PVIF_a(20\text{ years, }10\%)$$
$$= \$262.11 \times 8.5136$$
$$PVA_{20}(\$262.11) = \$2231.50.$$

Therefore, this technique of turning down the thermostats by 10 degrees during the colder part of the year results in cost savings with a present value of $2231.50.

PRESENT VALUE OF AN UNENDING ANNUITY OF COST SAVINGS

In many cases, the cost savings generated by an energy conservation project will be permanent. If these savings are the same amount each year, they form a sort of "unending" annuity. The present value of these cost savings is easy to calculate by using the following equation:

$$PVA_{unending}(\$M) = \frac{\$M}{r} \qquad \text{Equation 5}$$

where:

$PVA_{unending}(\$M)$ = Present value of an annuity of $M per year for every year in the future.
$M = Amount of the annuity.
r = Relevant percentage interest rate.

This technique can be useful in situations where nothing is purchased but the method of doing something is altered. It is helpful to look at the following example (Example G):

EXAMPLE G: PRESENT VALUE OF AN UNENDING ANNUITY.

Given: As the light bulbs in the halls of an office building burn out, they are to be replaced with lower wattage bulbs which will continue to provide sufficient illumination. This process will result in cost savings of $300 per year for each subsequent year. The interest rate is 7%.

Find: The present value of these cost savings.

Solution: These cost savings form an unending annuity of $300 per year. Therefore, Equation 5 is applicable:

$$PVA_{unending}(\$300) = \frac{\$300}{0.07}$$

$$PVA_{unending}(\$300) = \$4285.71.$$

The calculation of the present value of an unending annuity is possible because the cost savings have smaller present values the farther away in time they occur. At most interest rates (except very small ones) cost savings beyond 50 years will have very small present values compared to their original value. Therefore, depending on the interest rate, all cost savings that occur beyond some year in the future have present values that are almost zero and do not need to be considered.

PRESENT VALUE OF AN IRREGULAR FLOW OF COST SAVINGS

With an anuity, the same amount is received or spent every year. In many cases, this situation is not realistic since costs are constantly changing. The energy specialist must be able to deal with the situation where the flow of cost savings changes from one year to the next. This type of fluctuation, called an irregular flow of cost savings, is dealt with in the following manner: The present value is calculated by taking the sum of the present values of the savings associated with each year. This process is demonstrated by Example H.

EXAMPLE H: PRESENT VALUE OF AN IRREGULAR FLOW COST SAVINGS.

Given: A retail store occupies 2.4×10^4 ft^2 of floor space and is open for business 3720 h/yr (12 h/d, Monday through Saturday). For lighting, the store has 375 fixtures having four 40-W bulbs. Each fixture measures 2 by 4 feet. Illumination is 100 footcandles. To reduce electricity costs, the removal of two lamps from every other light fixture in the store has been recommended. This change would reduce illumination to 82.5 footcandles, which would maintain adequate illumination for store operation. The cost of electricity will be 3.4 cents per kWh for the next 5 years, at which time it will increase to 4 cents per kWh. The relevant interest rate is 10%.

Example H. Continued.

Find: The annual cost savings associated with this project, and the present value of these savings over the next 10 years.

Solution: $\frac{375}{2}$ = Number of fixtures where change is made

40 W per lamp x 2 lamps changed per fixture = 80 W saved per fixture where change is made.

Therefore,

$$\text{Watts Conserved} = \frac{375}{2}\ \text{fixtures} \times 80\ \text{W/fixture}$$

$$= 15{,}000\ \text{W.}$$

Since 1000 W = 1 kW, the savings can also be expressed as follows:

$$\frac{15{,}000}{1000} = 15\ \text{kW.}$$

Now, since the store is open 3720 h/yr, the amount of energy saved per year is given by the following:

$$15\ \text{kW} \times 3720\ \text{h} = 55{,}800\ \text{kWh.}$$

The cost savings for each of the first 5 years are given as follows:

$$55{,}800\ \frac{\text{kWh}}{\text{yr}} = \$0.034/\text{kWh} = \$1897.20/\text{yr.}$$

After the fifth year, when the price of electricity will increase, the annual cost savings will be as follows:

$$55{,}800\ \text{kWh} \times \$0.04/\text{kWh} = \$2322/\text{yr.}$$

Example H. Continued.

So, the savings of the project form an annuity of $1897.20/yr for 5 years, followed by another 5-year annuity of $2322/yr. The present value of these cost savings may be determined by finding the present value of the cost savings associated with each year (a series of fixed amounts) and then taking their sum. This process is shown in Table 6.

TABLE 6. PRESENT VALUE OF AN IRREGULAR FLOW OF COST SAVINGS.

Year	Cost Savings	PVIF (Data Table 2)	Present Value
1	$1897.20	0.9091	$1724.74
2	1897.20	0.8264	1567.85
3	1897.20	0.7513	1425.37
4	1897.20	0.6830	1295.79
5	1897.20	0.6209	1177.97
6	2322.00	0.5645	1310.77
7	2322.00	0.5132	1191.65
8	2322.00	0.4665	1083.21
9	2322.00	0.4241	984.76
10	2322.00	0.3855	895.13
		Total Present Value of Cost Savings =	$12657.24

COST ESCALATION AND PRESENT VALUE

The most likely instance of an irregular flow of cost savings is when energy costs increase, resulting in a greater potential cost savings each year. In this situation the cost savings of each year in the future are computed, using the cost escalation method presented in Module EE-02, "Financial Parameters of Energy Economics." Then the present value of each year's cost savings are added together (in the same manner they were added in Example H of this module). The following example (Example I) should make this procedure easier to understand:

EXAMPLE I: COST ESCALATION AND PRESENT VALUE.

Given: Energy conservation measures will result in savings of 3.6×10^8 ft^3 of natural gas per year for 4 years. The price of natural gas is now \$2.50/M ft^3 and is expected to rise at a rate of 6% per year for the next 4 years. The interest rate is 12%.

Find: The cost savings associated with each of the next 4 years and the total present value of the savings.

Solution: Cost of natural gas after 1 year = Cost of natural gas now = FVIF (1 year, 6%) (obtained from Data Table 1)

= \$2.50 x 1.0600

= \$2.65/M ft^3.

Cost savings of first year = \$2.65/M ft^3 x 3.6×10^8 ft^3

= \$954.

Example I. Continued.

Cost of natural gas after 2 years = \$2.50 x 1.1236
= \$2.81/M ft^3.

Cost savings of second year = \$2.81/M ft^3 x 3.6 x 10^8 ft^3
= \$1011.60.

Cost of natural gas after 3 years = \$2.50 x 1.1910
= \$2.98/M ft^3.

Cost savings of third year = \$2.98/M ft^3 x 360 M ft^3
= \$1072.80.

Cost of natural gas after 4 years = \$2.50 x 1.2625
= \$3.16/M ft^3.

Cost savings of fourth year = \$3.16/M ft^3 x 360 M ft^3
= \$1137.60.

The present value of the cost savings of each year is calculated and summed in Table 7.

TABLE 7. PRESENT VALUE OF COST SAVINGS.

Year	Cost Savings	x	PVIF (Data Table 2)	=	Present Value of Savings
1	\$ 954.00		0.8929		\$ 851.83
2	1011.60		0.7972		806.45
3	1072.80		0.7118		763.62
4	1137.60		0.6355		722.94
Total Present Value of the Cost Savings of the 4 Years				=	\$3144.84

CONCLUSION

The purpose of the techniques presented in this module is to give the student the ability to assess accurately the costs and cost savings associated with proposed energy projects. These techniques and their underlying principles are central to the types of analyses the energy specialist must perform.

EXERCISES

1. In a plant where the value of steam is $1.20 per million Btus, a leak estimated to be one-eighth of an inch in diameter was found in a steam tracing line operating at 100 psig. The steam loss was at an annual rate of 540 million Btus (Figure 1).

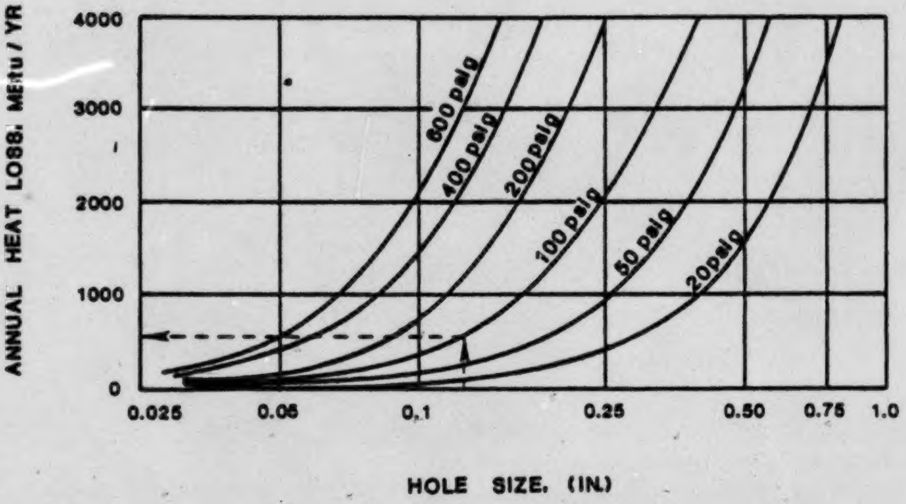

Figure 1. Heat Loss from Steam Leaks.

  a. Determine the annual savings associated with the repair of the leak.
  b. Assuming a 15-year life of repair and an interest rate of 8%, what is the present value of the savings associated with the repair of the steam leak?

2. The potential for saving energy by regularly monitoring boiler efficiency is illustrated by the detection of a deteriorated baffle in the boiler which had an estimated repair cost of $1500 but wasted fuel costing $40.50/d.

This manufacturing plant had oil-burning boilers producing approximately 100,000 pounds of steam per hour.

The boilers were modern and were equipped with wide-range, flow-metering, combustion control. The total systems were carefully set up, the combustion was tested by the manufacturer's representatives, and the installed boiler efficiency was determined.

A routine was set up to take integrated steam-flow and oil-flow readings and to compute the boiler efficiencies each day. A running chart of the daily efficiencies was made so that performance could be compared, at a glance, with the original efficiency. This procedure worked well for several years, with gradual degradation showing the need for periodically calling in the control manufacturer's service engineer for a combustion control tune-up.

During one period, the efficiency of one of the boilers, which generated 60,000 lb of steam per hour, began to decrease rapidly, dropping about 2% over a 2-week period. An analysis of other readings showed that the average flue-gas temperature for that boiler had deteriorated, causing the high flue-gas temperature and the resulting lower efficiency.

This boiler had a total daily steam flow of 1,440,000 lb, and each pound of steam absorbed 980 Btus of heat in the boiler. The total daily oil flow to this boiler was 12,700 gallons of oil, with a heating value of 139,300 Btu/gal. Thus, the boiler efficiency can be expressed as follows:

Efficiency, %

$$= \frac{\text{heat absorbed}}{\text{heat input}} \times 100\%$$

$$= \frac{1{,}440{,}000 \text{ lb steam/d} \times 980 \text{ Btu/lb}}{12{,}700 \text{ gal oil} \times 139{,}300 \text{ Btu/gal}} \times 100\%$$

$$= 79.8\%.$$

Rate of heat loss at a decrease of 2% in efficiency was as follows:

Heat input at 79.8% efficiency

= 12,700 gal/d x 139,300 Btu/gal

= 1769 MBtu/d.

Heat input at 77.8% efficiency

$$= \frac{1{,}440{,}000 \text{ lb/d} \times 980 \text{ Btu-1 lb}}{0.778}$$

= 1814 MBtu/d.

Heat loss

= 45 MBtu/d.

At a fuel cost of $0.90/MBtu, the loss is as follows:

Cost of loss

= 45 MBtu/d x $0.90/MBtu

= $40.50/day.

Estimated repair cost for baffle = $1500.

SUGGESTED ACTION

Establish a program to monitor boiler efficiency as a tool to control energy waste.

SOURCE: An equipment manufacturer.

a. Assume that the boiler is in operation 365 d/yr and that the life of the repair is 1 year. The interest rate is 10%. Should the repair be done?

b. Suppose that the cost of fuel is escalating at 1% per month. Calculate the present value of the cost savings from 1 year of operation.

c. If the minimum life of the repair is 2 years, what is the maximum amount the manufacturing plant should be willing to pay for repairing the baffle?

DATA TABLE 1. FUTURE VALUE OF $1 (FVIF).

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1200	1.1400	1.1500	1.1600	1.1800	1.2000	1.2400	1.2800	1.3200	1.3600
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2544	1.2996	1.3225	1.3456	1.3924	1.4400	1.5376	1.6384	1.7424	1.8496
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.4049	1.4815	1.5209	1.5609	1.6430	1.7280	1.9066	2.0972	2.3000	2.5155
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5735	1.6890	1.7490	1.8106	1.9388	2.0736	2.3642	2.6844	3.0360	3.4210
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.7623	1.9254	2.0114	2.1003	2.2878	2.4883	2.9316	3.4360	4.0075	4.6526
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.9738	2.1950	2.3131	2.4364	2.6996	2.9860	3.6352	4.3980	5.2899	6.3275
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.2107	2.5023	2.6600	2.8262	3.1855	3.5832	4.5077	5.6295	6.9826	8.6054
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.4760	2.8526	3.0590	3.2784	3.7589	4.2998	5.5895	7.2058	9.2170	11.703
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.7731	3.2519	3.5179	3.8030	4.4355	5.1598	6.9310	9.2234	12.166	15.916
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	3.1058	3.7072	4.0456	4.4114	5.2338	6.1917	8.5944	11.805	16.059	21.646
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.4785	4.2262	4.6524	5.1173	6.1759	7.4301	10.657	15.111	21.198	29.439
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.8960	4.8179	5.3502	5.9360	7.2876	8.9161	13.214	19.342	27.982	40.037
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	4.3635	5.4924	6.1528	6.8858	8.5994	10.699	16.386	24.758	36.937	54.451
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.8871	6.2613	7.0757	7.9875	10.147	12.839	20.319	31.691	48.756	74.053
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	5.4736	7.1379	8.1371	9.2655	11.973	15.407	25.195	40.564	64.358	100.71
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	6.1304	8.1372	9.3576	10.748	14.129	18.488	31.242	51.923	84.953	136.96
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	6.8660	9.2765	10.761	12.467	16.672	22.186	38.740	66.461	112.13	186.27
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	7.6900	10.575	12.375	14.462	19.673	26.623	48.038	85.070	148.02	253.33
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	8.6128	12.055	14.231	16.776	23.214	31.948	59.567	108.89	195.39	344.53
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	9.6463	13.743	16.366	19.460	27.393	38.337	73.864	139.37	257.91	468.57
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	10.803	15.667	18.821	22.574	32.323	46.005	91.591	178.40	340.44	637.26
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	12.100	17.861	21.644	26.186	38.142	55.205	113.57	228.35	449.39	866.67
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	13.552	20.361	24.891	30.376	45.007	66.247	140.83	292.30	593.19	1178.6
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	15.178	23.212	28.625	35.236	53.108	79.496	174.63	374.14	783.02	1602.9
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.834	17.000	26.461	32.918	40.874	62.668	95.396	216.54	478.90	1033.5	2180.0
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.918	19.040	30.166	37.856	47.414	73.948	114.47	268.51	612.99	1364.3	2964.9
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.245	13.110	21.324	34.389	43.535	55.000	87.259	137.37	332.95	784.63	1800.9	4032.2
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.167	14.421	23.883	39.204	50.065	63.800	102.96	164.84	412.86	1004.3	2377.2	5483.8
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.172	15.863	26.749	44.693	57.575	74.008	121.50	197.81	511.95	1285.5	3137.9	7458.0
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.062	13.267	17.449	29.959	50.950	66.211	85.849	143.37	237.37	634.81	1645.5	4142.0	10143.
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.285	14.974	21.724	31.409	45.259	93.050	188.88	267.86	378.72	750.37	1469.7	5455.9	19426.	66520.	*
50	1.6446	2.6916	4.3839	7.1067	11.467	18.420	29.457	46.901	74.357	117.39	289.00	700.23	1083.6	1670.7	3927.3	9100.4	46890.	*	*	*
60	1.8167	3.2810	5.8916	10.519	18.679	32.987	57.946	101.25	176.03	304.48	897.59	2595.9	4383.9	7370.1	20555.	56347.	*	*	*	*

*FVIF > 99,999

DATA TABLE 2. PRESENT VALUE OF $1 (PVIF).

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174	.9091	.8929	.8772	.8696	.8621	.8475	.8333	.8065	.7813	.7576	.7353
2	.9803	.9612	.9426	.9246	.9070	.8900	.8734	.8573	.8417	.8264	.7972	.7695	.7561	.7432	.7182	.6944	.6504	.6104	.5739	.5407
3	.9706	.9423	.9151	.8890	.8638	.8396	.8163	.7938	.7722	.7513	.7118	.6750	.6575	.6407	.6086	.5787	.5245	.4768	.4348	.3975
4	.9610	.9238	.8885	.8548	.8227	.7921	.7629	.7350	.7084	.6830	.6355	.5921	.5718	.5523	.5158	.4823	.4230	.3725	.3294	.2923
5	.9515	.9057	.8626	.8219	.7835	.7473	.7130	.6806	.6499	.6209	.5674	.5194	.4972	.4761	.4371	.4019	.3411	.2910	.2495	.2149
6	.9420	.8880	.8375	.7903	.7462	.7050	.6663	.6302	.5963	.5645	.5066	.4556	.4323	.4104	.3704	.3349	.2751	.2274	.1890	.1580
7	.9327	.8706	.8131	.7599	.7107	.6651	.6227	.5835	.5470	.5132	.4523	.3996	.3759	.3538	.3139	.2791	.2218	.1776	.1432	.1162
8	.9235	.8535	.7894	.7307	.6768	.6274	.5820	.5403	.5019	.4665	.4039	.3506	.3269	.3050	.2660	.2326	.1789	.1388	.1085	.0854
9	.9143	.8368	.7664	.7026	.6446	.5919	.5439	.5002	.4604	.4241	.3606	.3075	.2843	.2630	.2255	.1938	.1443	.1084	.0822	.0628
10	.9053	.8203	.7441	.6756	.6139	.5584	.5083	.4632	.4224	.3855	.3220	.2697	.2472	.2267	.1911	.1615	.1164	.0847	.0623	.0462
11	.8963	.8043	.7224	.6496	.5847	.5268	.4751	.4289	.3875	.3505	.2875	.2366	.2149	.1954	.1619	.1346	.0938	.0662	.0472	.0340
12	.8874	.7885	.7014	.6246	.5568	.4970	.4440	.3971	.3555	.3186	.2567	.2076	.1869	.1685	.1372	.1122	.0757	.0517	.0357	.0250
13	.8787	.7730	.6810	.6006	.5303	.4688	.4150	.3677	.3262	.2897	.2292	.1821	.1625	.1452	.1163	.0935	.0610	.0404	.0271	.0184
14	.8700	.7579	.6611	.5775	.5051	.4423	.3878	.3405	.2992	.2633	.2046	.1597	.1413	.1252	.0985	.0779	.0492	.0316	.0205	.0135
15	.8613	.7430	.6419	.5553	.4810	.4173	.3624	.3152	.2745	.2394	.1827	.1401	.1229	.1079	.0835	.0649	.0397	.0247	.0155	.0099
16	.8528	.7284	.6232	.5339	.4581	.3936	.3387	.2919	.2519	.2176	.1631	.1229	.1069	.0930	.0708	.0541	.0320	.0193	.0118	.0073
17	.8444	.7142	.6050	.5134	.4363	.3714	.3166	.2703	.2311	.1978	.1456	.1078	.0929	.0802	.0600	.0451	.0258	.0150	.0089	.0054
18	.8360	.7002	.5874	.4936	.4155	.3503	.2959	.2502	.2120	.1799	.1300	.0946	.0808	.0691	.0508	.0376	.0208	.0118	.0068	.0039
19	.8277	.6864	.5703	.4746	.3957	.3305	.2765	.2317	.1945	.1635	.1161	.0829	.0703	.0596	.0431	.0313	.0168	.0092	.0051	.0029
20	.8195	.6730	.5537	.4564	.3769	.3118	.2584	.2145	.1784	.1486	.1037	.0728	.0611	.0514	.0365	.0261	.0135	.0072	.0039	.0021
25	.7798	.6095	.4776	.3751	.2953	.2330	.1842	.1460	.1160	.0923	.0588	.0378	.0304	.0245	.0160	.0105	.0046	.0021	.0010	.0005
30	.7419	.5521	.4120	.3083	.2314	.1741	.1314	.0994	.0754	.0573	.0334	.0196	.0151	.0116	.0070	.0042	.0016	.0006	.0002	.0001
40	.6717	.4529	.3066	.2083	.1420	.0972	.0668	.0460	.0318	.0221	.0107	.0053	.0037	.0026	.0013	.0007	.0002	.0001	*	*
50	.6080	.3715	.2281	.1407	.0872	.0543	.0339	.0213	.0134	.0085	.0035	.0014	.0009	.0006	.0003	.0001	*	*	*	*
60	.5504	.3048	.1697	.0951	.0535	.0303	.0173	.0099	.0057	.0033	.0011	.0004	.0002	.0001	*	*	*	*	*	*

*The factor is zero to four decimal places.

DATA TABLE 3. FUTURE VALUE OF AN ANNUITY OF $1 ($FVIF_a$).

Number of Periods	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.259	10.636	11.028	11.435
9	9.3685	9.7546	10.159	10.582	11.026	11.491	11.978	12.487	13.021	13.579
10	10.462	10.949	11.463	12.006	12.577	13.180	13.816	14.486	15.192	15.937
11	11.566	12.168	12.807	13.486	14.206	14.971	15.783	16.645	17.560	18.531
12	12.682	13.412	14.192	15.025	15.917	16.869	17.888	18.977	20.140	21.384
13	13.809	14.680	15.617	16.626	17.713	18.882	20.140	21.495	22.953	24.522
14	14.947	15.973	17.086	18.291	19.598	21.015	22.550	24.214	26.019	27.975
15	16.096	17.293	18.598	20.023	21.578	23.276	25.129	27.152	29.360	31.772
16	17.257	18.639	20.156	21.824	23.657	25.672	27.888	30.324	33.003	35.949
17	18.430	20.012	21.761	23.697	25.840	28.212	30.840	33.750	36.973	40.544
18	19.614	21.412	23.414	25.645	28.132	30.905	33.999	37.450	41.301	45.599
19	20.810	22.840	25.116	27.671	30.539	33.760	37.379	41.446	46.018	51.159
20	22.019	24.297	26.870	29.778	33.066	36.785	40.995	45.762	51.160	57.275
21	23.239	25.783	28.676	31.969	35.719	39.992	44.865	50.422	56.764	64.002
22	24.471	27.299	30.536	34.248	38.505	43.392	49.005	55.456	62.873	71.402
23	25.716	28.845	32.452	36.617	41.430	46.995	53.436	60.893	69.531	79.543
24	26.973	30.421	34.426	39.082	44.502	50.815	58.176	66.764	76.789	88.497
25	28.243	32.030	36.459	41.645	47.727	54.864	63.249	73.105	84.700	98.347
26	29.525	33.670	38.553	44.311	51.113	59.156	68.676	79.954	93.323	109.18
27	30.820	35.344	40.709	47.084	54.669	63.705	74.483	87.350	102.72	121.09
28	32.129	37.051	42.930	49.967	58.402	68.528	80.697	95.338	112.96	134.20
29	33.450	38.792	45.218	52.966	62.322	73.639	87.346	103.96	124.13	148.63
30	34.784	40.568	47.575	56.084	66.438	79.058	94.460	113.28	136.30	164.49
40	48.886	60.402	75.401	95.025	120.79	154.76	199.63	259.05	337.88	442.59
50	64.463	84.579	112.79	152.66	209.34	290.33	406.52	573.76	815.08	1163.9
60	81.669	114.05	163.05	237.99	353.58	533.12	813.52	1253.2	1944.7	3034.8

Number of Periods	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.1200	2.1400	2.1500	2.1600	2.1800	2.2000	2.2400	2.2800	2.3200	2.3600
3	3.3744	3.4396	3.4725	3.5056	3.5724	3.6400	3.7776	3.9184	4.0624	4.2096
4	4.7793	4.9211	4.9934	5.0665	5.2154	5.3680	5.6842	6.0156	6.3624	6.7251
5	6.3528	6.6101	6.7424	6.8771	7.1542	7.4416	8.0484	8.6999	9.3983	10.146
6	8.1152	8.5355	8.7537	8.9775	9.4420	9.9299	10.980	12.135	13.405	14.798
7	10.089	10.730	11.066	11.413	12.141	12.915	14.615	16.533	18.695	21.126
8	12.299	13.232	13.726	14.240	15.327	16.499	19.122	22.163	25.678	29.731
9	14.775	16.085	16.785	17.518	19.085	20.798	24.712	29.369	34.895	41.435
10	17.548	19.337	20.303	21.321	23.521	25.958	31.643	38.592	47.061	57.351
11	20.654	23.044	24.349	25.732	28.755	32.150	40.237	50.398	63.121	78.998
12	24.133	27.270	29.001	30.850	34.931	39.580	50.894	65.510	84.320	108.43
13	28.029	32.088	34.351	36.786	42.218	48.496	64.109	84.852	112.30	148.47
14	32.392	37.581	40.504	43.672	50.818	59.195	80.496	109.61	149.23	202.92
15	37.279	43.842	47.580	51.659	60.965	72.035	100.81	141.30	197.99	276.97
16	42.753	50.980	55.717	60.925	72.939	87.442	126.01	181.86	262.35	377.69
17	48.883	59.117	65.075	71.673	87.068	105.93	157.25	233.79	347.30	514.66
18	55.749	68.394	75.836	84.140	103.74	128.11	195.99	300.25	459.44	700.93
19	63.439	78.969	88.211	98.603	123.41	154.74	244.03	385.32	607.47	954.27
20	72.052	91.024	102.44	115.37	146.62	186.68	303.60	494.21	802.86	1298.8
21	81.698	104.76	118.81	134.84	174.02	225.02	377.46	633.59	1060.7	1767.3
22	92.502	120.43	137.63	157.41	206.34	271.03	469.05	811.99	1401.2	2404.6
23	104.60	138.29	159.27	183.60	244.48	326.23	582.62	1040.3	1850.6	3271.3
24	118.15	158.65	184.16	213.97	289.49	392.48	723.46	1332.6	2443.8	4449.9
25	133.33	181.87	212.79	249.21	342.60	471.98	898.09	1706.8	3226.8	6052.9
26	150.33	208.33	245.71	290.08	405.27	567.37	1114.6	2185.7	4260.4	8233.0
27	169.37	238.49	283.56	337.50	479.22	681.85	1383.1	2798.7	5624.7	11197.9
28	190.69	272.88	327.10	392.50	566.48	819.22	1716.0	3583.3	7425.6	15230.2
29	214.58	312.09	377.16	456.30	669.44	984.06	2128.9	4587.6	9802.9	20714.1
30	241.33	356.78	434.74	530.31	790.94	1181.8	2640.9	5873.2	12940.	28172.2
40	767.09	1342.0	1779.0	2360.7	4163.2	7343.8	22728.	69377.	*	*
50	2400.0	4994.5	7217.7	10435.	21813.	45497.	*	*	*	*
60	7471.6	18535.	29219.	46057.	*	*	*	*	*	*

*FVIFA > 99,999

DATA TABLE 4. PRESENT VALUE OF AN ANNUITY OF $1 ($PVIF_a$).

Number of payments	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.8929	0.8772	0.8696	0.8621	0.8475	0.8333	0.8065	0.7813	0.7576
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.6901	1.6467	1.6257	1.6052	1.5656	1.5278	1.4568	1.3916	1.3316
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4018	2.3216	2.2832	2.2459	2.1743	2.1065	1.9813	1.8684	1.7663
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.0373	2.9137	2.8550	2.7982	2.6901	2.5887	2.4043	2.2410	2.0957
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6048	3.4331	3.3522	3.2743	3.1272	2.9906	2.7454	2.5320	2.3452
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.1114	3.8887	3.7845	3.6847	3.4976	3.3255	3.0205	2.7594	2.5342
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.5638	4.2883	4.1604	4.0386	3.8115	3.6046	3.2423	2.9370	2.6775
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	4.9676	4.6389	4.4873	4.3436	4.0776	3.8372	3.4212	3.0758	2.7860
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.3282	4.9464	4.7716	4.6065	4.3030	4.0310	3.5655	3.1842	2.8681
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.6502	5.2161	5.0188	4.8332	4.4941	4.1925	3.6819	3.2689	2.9304
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	5.9377	5.4527	5.2337	5.0286	4.6560	4.3271	3.7757	3.3351	2.9776
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.1944	5.6603	5.4206	5.1971	4.7932	4.4392	3.8514	3.3868	3.0133
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.4235	5.8424	5.5831	5.3423	4.9095	4.5327	3.9124	3.4272	3.0404
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.6282	6.0021	5.7245	5.4675	5.0081	4.6106	3.9616	3.4587	3.0609
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	6.8109	6.1422	5.8474	5.5755	5.0916	4.6755	4.0013	3.4834	3.0764
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	6.9740	6.2651	5.9542	5.6685	5.1624	4.7296	4.0333	3.5026	3.0882
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.1196	6.3729	6.0472	5.7487	5.2223	4.7746	4.0591	3.5177	3.0971
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.2497	6.4674	6.1280	5.8178	5.2732	4.8122	4.0799	3.5294	3.1039
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.3658	6.5504	6.1982	5.8775	5.3162	4.8435	4.0967	3.5386	3.1090
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.4694	6.6231	6.2593	5.9288	5.3527	4.8696	4.1103	3.5458	3.1129
25	22.0232	19.5235	17.4131	15.6221	14.0939	12.7834	11.6536	10.6748	9.8226	9.0770	7.8431	6.8729	6.4641	6.0971	5.4669	4.9476	4.1474	3.5640	3.1220
30	25.8077	22.3965	19.6004	17.2920	15.3725	13.7648	12.4090	11.2578	10.2737	9.4269	8.0552	7.0027	6.5660	6.1772	5.5168	4.9789	4.1601	3.5693	3.1242
40	32.8347	27.3555	23.1148	19.7928	17.1591	15.0463	13.3317	11.9246	10.7574	9.7791	8.2438	7.1050	6.6418	6.2335	5.5482	4.9966	4.1659	3.5712	3.1250
50	39.1961	31.4236	25.7298	21.4822	18.2559	15.7619	13.8007	12.2335	10.9617	9.9148	8.3045	7.1327	6.6605	6.2463	5.5541	4.9995	4.1666	3.5714	3.1250
60	44.9550	34.7609	27.6756	22.6235	18.9293	16.1614	14.0392	12.3766	11.0480	9.9672	8.3240	7.1401	6.6651	6.2492	5.5553	4.9999	4.1667	3.5714	3.1250

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GLOSSARY

Annuity: The recurring receipt or expenditure of a constant dollar amount over several consecutive time periods.

Compound sum: The future value a dollar amount will grow to if invested now at a specified interest rate.

Cost savings: A decrease in future operating costs resulting from a specific action taken now.

Future value interest factor of an annuity: A number that yields the future amount an annuity will grow to if invested at a specified rate.

Irregular flow of cost savings: A change in the flow of cost savings from one year to the next.

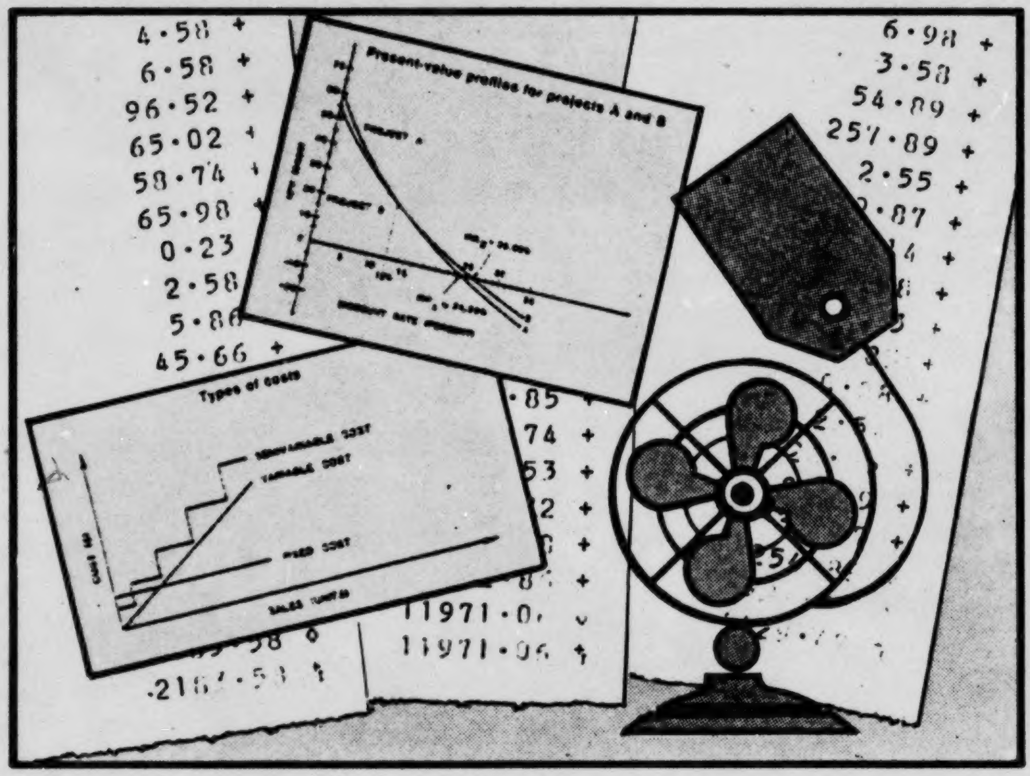
Present value of an annuity: The discounted value of a constant stream of funds at a specified interest rate.

Unending annuity: A constant stream of funds that has an unlimited life.

ENERGY TECHNOLOGY

CONSERVATION AND USE

ENERGY ECONOMICS



EE-04

ECONOMICS OF ENERGY ALTERNATIVES

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INTRODUCTION

In many cases, money must be spent to implement/or continue an energy conservation program, but the exact amount of money required can vary, depending on the size of the project and other factors. For instance, buying and installing insulation in a large home would be a more costly project than repairing several small leaks in a natural gas line.

Since energy (fuels) costs money, anything that saves energy saves money. The more money an energy conservation project costs, the more money it must save and, therefore, the more energy it must save. The important consideration concerning any energy conservation project is how its cost savings compare with its costs in present-value terms. An energy specialist, therefore, must be able to determine accurately the cost of an energy project and the cost savings it will generate, and then convert these figures to present value. By now, the student should be familiar with the calculation of present value when costs and cost savings are given.

This module explains several concepts, techniques, and factors which can be helpful in the accurate determination of costs and cost savings. Present-value techniques are only as good as the cost estimates on which they are based; therefore, the energy specialist must be concerned with the accuracy of cost and cost-savings figures.

In this module, the student is introduced to several factors that can affect the actual level of the costs and cost savings of energy projects. Often the actual level is not what it initally appears to be. The process of borrowing money to finance an energy project is discussed, and the impact on costs is demonstrated. Taxes and their effect on costs and cost savings are explored. Finally, the concept of life-cycle costing is presented. In all cases, the student is introduced to techniques that adjust for these factors in cost and cost-saving determinations.

PREREQUISITES

The student should have a good understanding of basic algebraic functions and should have completed Modules EE-02, and EE-03 of Energy Economics.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Define the following terms:
   a. Investment.
   b. Down payment.
   c. Installments.
   d. Outstanding balance.
   e. Tax deduction.
   f. Marginal tax rate.
   g. Tax credit.
   h. Life of investment — costing period.
   i. Engineering, design, and development costs.
   j. Product costs.
   k. Operating costs.
   l. Life-cycle cost.
2. Given the appropriate information, compute the following:
   a. The dollar effect of a tax deduction.
   b. The dollar effect of a tax credit.

3. Explain the effect of tax deductions and credits on the following:
    a. Costs.
    b. Cost savings.
4. Determine how much of an installment payment applies to each of the following:
    a. Interest.
    b. Reduction of principal.
5. Classify the costs of an energy project into the appropriate life-cycle costing categories, and use the information to calculate the total cost of owning and operating an object or system.
6. Analyze the way in which the costs of an energy project relate to the cost savings it generates in present-value terms.
7. Describe the use of marginal analysis in the evaluation of energy projects.

FINANCES OF AN INVESTMENT

Whenever money is spent by an individual or by an organization on a project (system) or a product with the purpose of reducing costs or otherwise making money, the object of that expenditure is called an investment. The process of purchasing and installing the system or product is called making an investment. The size of the investment – the amount of money spent – varies widely, depending on the nature of the particular project. When the investment is large, often the person or business will borrow the money that is needed because there is not enough "idle cash" (money in the bank that is not being used for anything else) available to purchase the desired equipment or system. Obtaining money to pay for an investment in energy conservation is called financing the investment. Money for investment is normally borrowed from a bank – although other organizations, such as savings and loan companies and insurance companies, will also lend money. It should be noted, however, that financing is often obtained from internal sources – money already in the possession of the business or individual.

When money is borrowed, interest must be paid to the lender. The interest is "rent" that is paid by the borrower for the use of the lender's money. The amount of interest paid per year is usually some percentage of the amount borrowed. This percentage is called the interest rate. A loan is usually paid back a little at a time – a certain amount per month or per year. The total amount paid back will be more than the amount borrowed because of the interest the borrower must pay. The process of repaying a loan is shown in Figure 1.

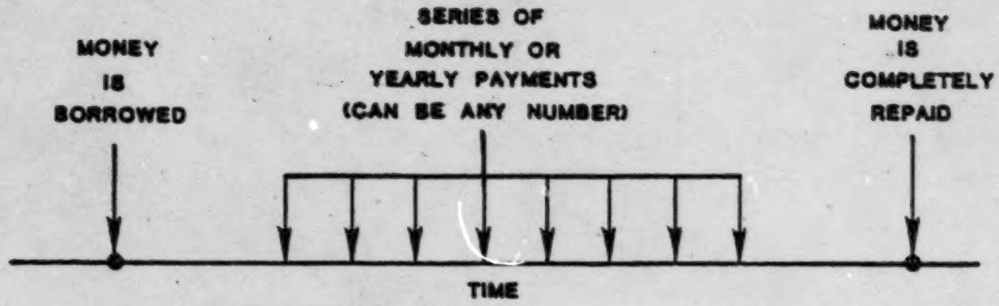
MONEY IS BORROWED
SERIES OF MONTHLY OR YEARLY PAYMENTS (CAN BE ANY NUMBER)
MONEY IS COMPLETELY REPAID
TIME

Figure 1. How a Loan is Repaid.

In most cases only part of the money needed to begin an energy conservation project is borrowed. Usually, part is paid initially and the remainder is borrowed. When this happens, the initial amount paid is called the down payment, and the payments to be made later are called installments. Each installment consists of two parts: (1) the interest charge, and (2) the amount paid to reduce the total amount borrowed. The total amount borrowed is called the principal. Thus, the following equation can be stated:

$$\text{Amount of installment} = \text{Interest charge} + \text{Amount paid to reduce the principal} \qquad \text{Equation 1}$$

The amount of the loan still to be repaid at any one time is called the outstanding balance of the loan. The interest charge portion of an installment is given by the following equation:

$$\text{Interest charge portion of installment} = \text{Outstanding balance when payment is made} \times \text{Interest rate} \qquad \text{Equation 2}$$

After the interest charge has been determined, the remaining amount is used to reduce the outstanding balance.

A series of installments can take one of two forms:

- The outstanding balance is reduced by the same amount with every payment. This means the interest charge will be different each month, so the total payment will be different each month.
- The total amount of the payment each month is always the same, so both the interest charge and the outstanding balance reduction will change.

The next two examples (Examples A and B) illustrate both forms of repayment.

EXAMPLE A: OUTSTANDING BALANCE REDUCED BY THE SAME AMOUNT.

Given: Three thousand dollars is borrowed to pay for home insulation. The interest rate is 10%. The outstanding balance of the loan is to be reduced by $1,000 each year for 3 years.

Find: The interest charge and total installment for each of the next 3 years.

Example A. Continued.

Solution: First year interest charge = Outstanding balance when payment is made x Interest rate

= \$3,000 x 0.10

= \$300.

Amount of first installment = Interest charge + Amount principal is reduced

= \$300 + \$1,000

= \$1,300.

The outstanding balance at the end of the first year is \$2,000.

Second year interest charge = \$2,000 x 0.10

= \$200.

Amount of second installment = \$200 + \$1,000

= \$1,200.

The outstanding balance at the end of the second year is \$1,000.

Third year interest charge = \$1,000 x 0.10

= \$100.

Amount of third installment = \$100 + \$1,000

= \$1,000.

The outstanding balance at the end of the third year is \$0; therefore, the loan has been repaid.

EXAMPLE B: CONSTANT INSTALLMENT AMOUNT.

Given: Three thousand dollars is borrowed to pay for home insulation. The interest rate is 10%. The total payment per year cannot be more than $1,000.

Find: The interest charge portion of each installment, and how long it will take for the loan to be repaid in full.

Solution: First year interest charge = $3,000 x 0.10
= $300.

Amount the outstanding balance is reduced = $1,000 - $300
= $700.

The outstanding balance at the end of the first year is $3,000 - $700 = $2,300.

Second year interest charge = $2,300 x 0.10
= $230.

Amount the outstanding balance is reduced = $1,000 - $230
= $770.

The outstanding balance at the end of the second year is $2,300 - $770 = $1,530.

Third year interest charge = $1,530 x 0.10
= $153.

Example B. Continued.

Amount the outstanding balance is reduced = \$1,000 - \$153
= \$847.

The outstanding balance at the end of the third year is \$683.

Fourth year interest charge = \$683 x 0.10
= \$68.30.

The outstanding balance can be reduced to zero with one installment:

Amount of Installment = Interest charge + Amount principal is reduced
= \$68.30 + \$683
= \$751.30.

Thus, the loan can be repaid with three \$1,000 installments, followed by an installment of \$751.30. The loan would take 4 years to repay.

It is obvious from the calculations given in Examples A and B that the form of repayment can make a substantial difference in the amount of money that is paid back per year and in the length of time the payments must be made. It is important to recognize that these payments are "costs" of an energy conservation project. Each of these costs occurs at some point in time, and the exact time each payment is made is very important because of the time value of money. The interest that is paid on the borrowed money is a cost just like the actual purchase price of the system or object. For this reason, the energy specialist must understand the borrowing and lending process.

TAXES

Taxes can have an important effect on costs because taxes apply to different things in different ways. For instance, suppose an individual purchases and installs a solar water-heating system with borrowed money. Personal income (money earned) will be used to pay the installments until the loan is repaid. These wages are subject to federal income tax, that is, a certain percentage of the wages this person earns must be paid to the federal government. However, when part of this income is spent on certain items, taxes do not have to be paid on that particular part. This particular part of the income is deductible. The amount of tax that must be paid can be stated as a percentage of the income subject to tax. This is called the tax rate. A person who pays a certain tax rate is said to be in that particular tax bracket. For instance, if 30% of a person's income is subject to tax, then that person is said to be in the 30% tax bracket.

Example C illustrates the concept of the average tax bracket. With federal income tax, however, not all income is taxed equally; greater amounts of income are taxed at higher rates.

EXAMPLE C: TAXES, TAX BRACKETS, AND DEDUCTIBLE INCOME.

Given: An individual is in the 25% tax bracket and has earned $20,000. Of that income, $5,000 was spent on deductible items.

Find: The amount of tax the individual must pay.

Solution:

$$\text{Amount subject to tax} = \text{Total income} + \text{Deductible expenses}$$

$$= \$20{,}000 - \$5{,}000$$

$$= \$15{,}000.$$

$$\text{Tax which must be paid} = \text{Amount subject to tax} \times \text{tax rate}$$

$$= \$15{,}000 \times 0.25$$

$$= \$3.750.$$

The tax rate that applies to the last dollar of income earned is called marginal tax rate. The marginal rate schedule for an individual could be similar to the one shown in Table 1.

TABLE 1. MARGINAL TAX RATES.

1st $2,000 - 15% ($0 through $2,000)
2nd $2,000 - 19% ($2,001 through $4,000)
3rd $2,000 - 22% ($4,001 through $6,000)
4th $2,000 - 25% ($6,001 through $8,000)
5th $2,000 - 28% ($8,001 through $10,000)
6th $2,000 - 32% ($10,001 through $12,000)

Therefore, if a person with a marginal rate schedule like the one in Table 1 earned $11,000 per year, the marginal tax rate on the last dollar earned would be 32%.

EXAMPLE D: THE MARGINAL TAX RATE.

Given:	An individual earns $10,500 per year and has a deductible expense of $1,500. (The rate schedule given in Table 1 is applicable.)
Find:	The after-tax cost of the $1,500 deduction.
Solution:	Of this deductible expense, $500 would have been taxable at 32%; therefore, the savings of that portion are $500 x 0.32 = $160. The other $1,000 of the expense would be taxable at 28%, so the savings of the deduction are $1,000 x 0.28 = $280. The total savings of the tax deduction are $160 + $280 = $440. Thus, the actual cost is $1,500 - $440 = $1,060.

Whenever the marginal rate schedule is available, it should be used instead of the average rate.

One important deductible expense is the interest paid on borrowed money. In other words, income used to pay interest is not subject to federal income tax. If an individual is in the 30% average tax bracket, then 30% of that person's income must be paid each year to the federal government. When this individual borrows money to finance an energy conservation project, the interest payments are deductible and will lower the actual cost of the project by reducing taxes paid. For example, if income that is used to pay interest were taxable, then, of every one dollar earned, only 70% could be used to pay interest, with the rest being paid for taxes. On the other hand, if the interest were not taxable, then 30% less could be earned to pay the interest, and not as much of the total amount earned would be spent on taxes. This applies to any expense that is deductible; therefore, taxes must be considered in order to reflect accurately the actual nature of costs and cost savings. Table 2 gives the cost savings of a payment of $500 being deductible when different tax rates are applicable.

TABLE 2. SAVINGS OF TAX DEDUCTION.

Relevant Tax Bracket	Amount That Must Be Earned If Payment Is Taxable	Amount If Not Taxable	Savings of Tax Deduction
10%	$ 555.56	$500	$ 55.56
20%	625.00	500	125.00
30%	714.29	500	214.29
40%	833.33	500	333.33
50%	1,000.00	500	500.00

Table 2 shows the obvious: that higher tax rates result in more savings when an expense is deductible. People in higher brackets normally must pay a higher tax; but when they do not have to pay the tax (that is, the expense is deductible), then the savings is greater.

CORPORATIONS AND TAXES

Tax situations vary with different individuals and organizations. For instance, some businesses — most large ones — are corporations. Corporations differ from some businesses in the manner in which they are orgainzed: the major feature is that they exist on their own, while other kinds of businesses are tied to one person or a group of people. All corporations pay taxes at the same rate — 46% on income beyond $50,000. (Most corporations earn quite a bit more than that every year.) This fact can make the effect of taxes on costs and cost savings much easier to determine.

Most business expenditures resulting from general operations are tax deductible. Consequently, the cost of business investment in energy conservation is partially offset by tax savings. The financial benefit of lower energy bills is also affected by taxes, as shown in Example E.

EXAMPLE E: CORPORATIONS AND TAXES.

Given: A corporation plans to set its thermostats higher in the summer months in its office building, and the savings in electricity will be $4,200 per year. The corporation pays 46% of its income in taxes.

Find: The actual savings per year of the project.

Example E. Continued.

Solution: Since the corporation pays 46% of its income in taxes, and since the electricity bill is a business expense and, therefore, tax deductible, when the corporation saves $4,200/yr in expenses, that $4,200 becomes taxable. Thus, the corporation's tax bill would increase by $4,200 x 0.46 = $1,932/yr. The actual savings is $4,200 - $1,932 = $2,268/yr.

Example E implies an easy way to compute the actual cost savings per year of an energy project when tax considerations are involved. This figure is given by the following formula:

$$\begin{matrix}\text{Actual cost} \\ \text{savings} \\ \text{per year}\end{matrix} = \begin{matrix}\text{Original} \\ \text{dollar} \\ \text{savings} \\ \text{per year}\end{matrix} \times \left(1 - \begin{matrix}\text{Applicable} \\ \text{tax rate}\end{matrix}\right) \qquad \text{Equation 3}$$

For instance, consider the situation in Example E as follows:

$$\begin{matrix}\text{Actual Cost} \\ \text{Savings} \\ \text{Per Year}\end{matrix} \begin{aligned} &= \$4,200 \times (1 - 0.46) \\ &= \$4,200 \times (0.54) \\ &= \$2,268/\text{yr.}\end{aligned}$$

Equation 3 can be used for businesses or individuals; it is essential, however, that the applicable tax rate be known.

It should be noted, also, that whenever a business undertakes an energy conservation project, the cost of the project is tax deductible. This makes the actual cost smaller. Actual cost can be computed in a method exactly like that used in Equation 3.

$$\text{Actual cost} = \text{Original cost} \times (1 - \text{Tax rate}) \qquad \text{Equation 4}$$

TAX CREDITS

Occasionaly the government wants to encourage spending on a particular item. When this is the case, a tax credit is often instituted. A tax credit is a reduction in the tax bill an individual or business must pay. For instance, if an individual's tax bill amounts to $4,000, and he or she qualifies for a $600 tax credit, then the amount of tax which must be paid is equal to $4,000 - $600 = $3,400. Tax credits can be expressed as either a dollar amount or a percentage of some other amount. Whenever a tax credit is associated with an energy project, it reduces the cost of the project, because money that otherwise would have been paid in taxes can be used for something else. This concept should become clearer with the following example (Example F):

EXAMPLE F: TAX CREDITS.

Given: An individual buys insulation to install in his or her home with personal money (that is, not borrowed). By insulating the home, this individual qualifies for a $300 income tax credit. The person's tax bill normally would be $3,500. The cost of the insulation is $1,000.

Find: The actual cost of the insulation.

Solution: Since the individual qualifies for a $300 tax credit, his or her tax bill will be reduced by $300. Although $1,000 is paid for the insulation, $300 less is paid in taxes. Therefore, the total amount spent is $1,000 - $300 = $700.

It should be clear at this point that tax considerations have a significant impact on both the costs and cost savings of energy projects. This particularly applies to businesses, where the costs are tax deductible and the cost savings are taxable. Often these considerations can completely change the way the costs of an energy project compare to its cost savings in present-value terms, and for this reason, they are very important.

EXAMPLE G: CORPORATIONS AND TAXES.

Given: A corporation purchases a microcomputer for \$15,000. The corporation is in the 46% tax bracket.

Find: The actual cost of the microcomputer to the company.

Solution: The microcomputer is a business expense and is, therefore, tax deductible. If income were not spent on it, the income would be taxable; therefore, 46% of the \$15,000 would have been spent anyway in the form of taxes. The actual cost is 54% of \$15,000. Using Equation 4, the following may be written:

$$\begin{aligned}\text{Actual cost} &= \$15{,}000 \times (1 - 0.46)\\ &= \$15{,}000 \times 0.54\\ &= \$8{,}100.\end{aligned}$$

LIFE OF AN INVESTMENT

Whenever the total cost and cost savings of an energy project are calculated, some knowledge is needed of how long the system or device will be working. An estimate of this figure can be obtained in a number of ways. Often the manufacturer of the equipment will have data regarding its expected life. The way in which the expected life of an investment is determined depends on the information available. It should be emphasized that it is much better to underestimate the life of an investment than overestimate it. If the equipment lasts longer than it was supposed to, then the result is additional cost savings. However, if a piece of equipment stops working before it is supposed to, then it will not generate all the cost savings it should. In the latter case, the cost of the project might actually be greater than the cost savings, making the project unadvisable.

LIFE-CYCLE COSTING

Life-cycle costing is a technique that is useful in determining the total cost of owning and using an energy conservation device – such as a microcomputer system. Energy conservation projects can have costs of many different kinds: purchase cost, installation cost, maintenance cost, labor cost, and so forth. Life-cycle costing takes all of the costs of an energy project and divides them into three categories: (1) engineering, design, and development costs; (2) total construction costs; and (3) operating costs. These categories are defined as follows:

1. Engineering, design, and development costs: This category includes all costs of making the project workable, including the costs of designing, producing, and applying the project to a given situation.
2. Construction: Includes the purchase price, installation, and all other costs needed to get the project into operation.
3. Operating costs: Once the system is installed, these are the costs of keeping it going. Such costs include maintenance, personnel, and so forth.

Several of these costs and the way they relate are shown in Figure 2.

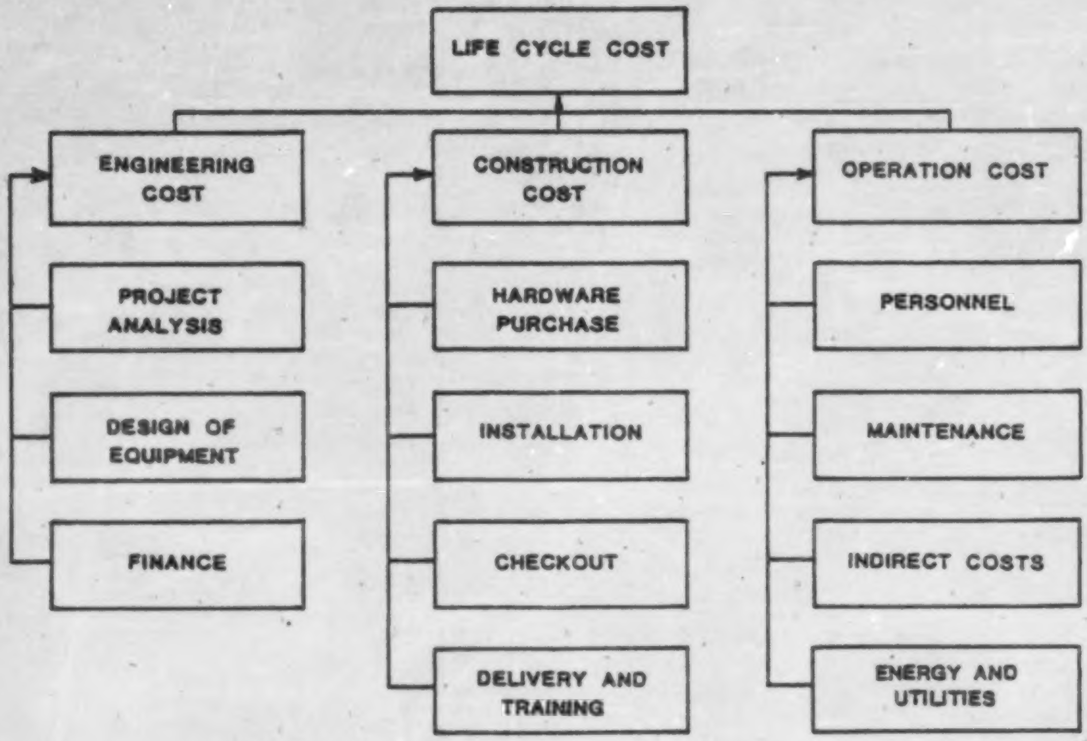

Figure 2. Elements of Life-Cycle Cost.

It should be pointed out that not all factors listed in Figure 2 apply to every energy conservation project. In fact, in some instances a system or device simply will be bought and installed; therefore, no engineering or design costs are involved. On the other hand, occasionally a system will have to be modified for application to a particular situation. In this case, some design cost would be involved. For example, a microcomputer system for thermostat control in a home could probably be installed with very little modification; but the installation of a solar-heating system to supplement a fuel-oil-powered system in the same home might involve some design work. A situation may also arise where there is some type of cost that does not fit into one of the categories shown in Figure 2. In all cases, however, the costs of an energy conservation project can be classified as follows: engineering costs, construction costs, or operating costs.

The process of life-cycle costing is useful because it provides a good method of comparing different projects which have the same function. A cooling system powered by electricity has the same function as one powered by natural gas — to provide a comfortable environment. The decision that is made concerning which kind of system to use will depend on the costs of each. The determination and analysis of these costs is not always an easy matter, however, because not all costs occur at the same time. In this case, present-value techniques become necessary. A system must be designed before it is installed, so the design cost will occur before the installation cost; a system must be installed before it is operated, so operation costs occur after installation costs. Figure 3 shows (1) when costs in the different categories occur and (2) how the magnitude of each type of cost changes over a period of time.

Several things should be noticed about Figure 3. There is some difference between the end of the costing period (the period of time over which the costs are calculated) and the end of the life cycle (when the product or system quits working). The costing period is determined by management, and, as mentioned earlier, it is better to underestimate the period (as is done in Figure 3) rather than to overestimate it. If the product will last until the end of the costing period, then it will perform all that is required of it to be

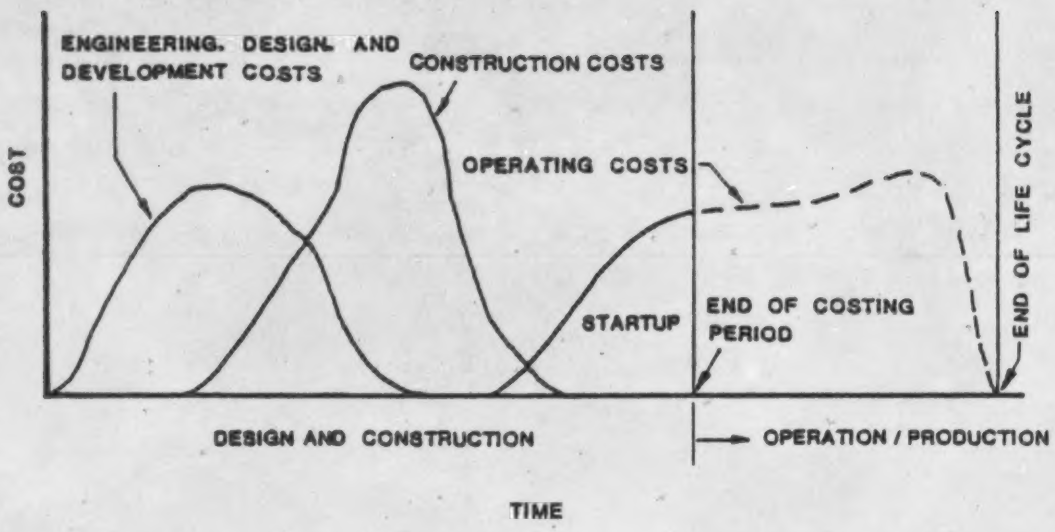

Figure 3. Stages of Life-Cycle Cost.

profitable – since the decision to implement it was based on the assumption that it would last until the end of the costing period. It should also be noted that not all costs in each category occur at one point in time. Operating costs occur as long as the system is used, and engineering and construction costs cover shorter time periods, tending to rise and fall in magnitude.

To do a life-cycle costing analysis, the following data are needed:

- Cost elements
- Operating profile – data regarding exactly what the system will do
- Utilization factors – to what extent the system will be used.
- Costs at current prices
- What costs are likely to be in the future – adjusting current prices for inflation with cost-escalation techniques
- Costing period on life

Once these data have been obtained, the life-cycle cost of a project can be calculated. Then its life-cycle cost can be compared to the life-cycle costs of other projects with the same goal or function. This comparison can be used to help determine what should be done. The technique of life-cycle costing is simply a method of incorporating all relevant costs into the decision-making process. Following is a detailed example of how life-cycle cost is determined:

EXAMPLE H: LIFE-CYCLE COSTING.

Given: An individual is considering the installation of a solar-energy system with 400 ft^2 of collector area in his or her home. The engineering and design cost is \$1,000 (to be incurred in the present), and the cost of the system installed is \$15/ft^2. The down payment would be 20% of the design and construction cost, with the remaining amount borrowed and paid back over a 20-year period at an interest rate of 9%. The annual payment is \$613, with the payment in the 20th year being slightly higher (\$633.22). The interest paid on the borrowed money is tax deductible, and the individual is in the 38% tax bracket. Maintenance costs for the system is \$100 for the first year and should be escalated at 6% per year over the 20-year life of the system. The discount rate that should be used in determing present value is 8%.

Example H. Continued.

Find: The present value of the total cost of owning the system.

Solution: Total cost associated with the present = $(\$15/\text{ft}^2 \times 400\ \text{ft}^2) + \$1{,}000$

= \$6,000 + \$1,000

= \$7,000.

Down payment = 20% of original cost

= 0.20 x \$7,000

= \$1,400.

Amount borrowed = Original cost - Down payment

= \$7,000 - \$1,400

= \$5,600.

This is the outstanding balance at the end of the first year just before the first payment of \$613 is made. To find the total cost of owning the system, the present value of the cost of each year in the future should be calculated. Each year in the future involves maintenance costs and repayment of part of the loan (the interest on which is tax deductible). Table 3 gives the calculation of the present value of the costs of each year. When this is added to the down payment, the total cost of owning the system is obtained (in present value terms).

TABLE 3. PRESENT VALUE OF SOLAR-ENERGY SYSTEM COSTS.

A	B	C	D	E	F	G	H	I
Year	Outstanding Balance at end of year	Interest Cost (B x 0.09)	Amt. applied to Principal ($613 - C)	Interest cost after tax C x (1 - 0.38)	Maintenance Cost escalated at 6%	Total Cost (D+E+F)	PVIF	Present Value
1	$5,600.00	$504.00	$109.00	$421.48	$100.00	$521.48	0.9259	$482.84
2	5,491.00	494.19	118.81	425.21	106.00	531.21	0.8573	455.41
3	5,372.19	483.50	129.50	429.27	112.36	541.63	0.7938	429.95
4	5,242.69	471.84	141.16	433.70	119.10	552.80	0.7350	406.31
5	5,101.53	459.14	153.86	438.53	126.25	564.78	0.6806	384.39
6	4,946.67	445.29	167.71	443.79	133.82	577.61	0.6302	364.01
7	4,778.96	430.11	182.89	449.56	141.85	591.41	0.5835	345.09
8	4,596.07	413.65	199.35	455.81	150.36	606.17	0.5403	327.51
9	4,396.72	395.70	217.30	462.63	159.38	622.01	0.5002	311.13
10	4,179.42	376.15	236.85	470.06	168.95	639.01	0.4632	295.99
11	3,942.57	354.83	258.17	478.16	179.08	657.24	0.4289	281.89
12	3,684.40	331.60	281.40	486.99	189.83	676.82	0.3971	268.77
13	3,403.00	306.27	306.73	496.62	201.22	697.84	0.3677	256.60
14	3,096.27	278.66	334.34	507.11	213.29	720.40	0.3405	245.30
15	2,761.93	248.57	364.43	518.54	226.09	744.63	0.3152	234.72
16	2,397.50	215.78	397.22	531.00	239.66	770.66	0.2919	224.96
17	2,000.28	180.03	432.97	544.59	254.04	798.63	0.2703	215.87
18	1,567.31	141.06	471.94	559.40	269.28	828.68	0.2503	207.42
19	1,095.37	98.58	514.42	575.54	285.43	860.97	0.2317	199.49
20	580.95	52.29	580.95	613.57	302.56	915.93	0.2145	196.47

Present value of future costs ...... $ 6,134.12

Down payment 1,400.00

Present value of the cost of owning and operating solar-energy system ...... $ 7,534.12

EXAMPLE I: DETERMINATION OF COST SAVINGS.

Given: In Example H, suppose that the backup energy source has a cost of $\$8/10^6$ Btu and is to be escalated at 10% per year for each of the 20 years. The annual energy load of the house is 152.83×10^6 Btu. If 400 ft^2 of collector area is installed, it will supply approximately 43% of the energy needs of the home. The discount rate is still 8%.

Find: The present value of the cost savings associated with the installation of this system.

Solution: Energy saved per year = 43% x Energy load

$$= 0.43 \times 152.83 \times 10^6 \text{ Btu}$$

$$= 65.72 \times 10^6 \text{ Btu.}$$

The cost savings per year can be computed by escalating the energy price for each of the next 20 years and then multiplying the price by the above figure. This is done in Table 4, where the present value is also taken.

TABLE 4. PRESENT VALUE OF COST SAVINGS.

A	B	C	D	E	F
Year	Price of Energy (10^6 Btu)	Energy Saved (10^6 Btu)	Annual Dollar Savings (B x C)	PVIF (8%)	Present Value of Savings (D x E)
1	$ 8.00	65.72	$ 525.76	0.9259	$ 486.80
2	8.80	65.72	578.34	0.8573	495.81
3	9.68	65.72	636.17	0.7938	504.99
4	10.65	65.72	699.92	0.7350	514.44
5	11.71	65.72	769.58	0.6806	523.47
6	12.88	65.72	846.47	0.6302	533.44
7	14.17	65.72	931.25	0.5835	543.38
8	15.59	65.72	1,024.57	0.5403	553.58
9	17.15	65.72	1,127.10	0.5002	563.78
10	18.86	65.72	1,239.48	0.4632	574.13
11	20.75	65.72	1,363.69	0.4289	584.89
12	22.82	65.72	1,499.73	0.3971	595.54
13	25.11	65.72	1,650.23	0.3677	606.79
14	27.62	65.72	1,815.19	0.3405	618.07
15	30.38	65.72	1,996.57	0.3152	629.32
16	33.42	65.72	2,196.36	0.2919	641.12
17	36.76	65.72	2,415.87	0.2703	653.01
18	40.44	65.72	2,657.72	0.2502	664.96
19	44.48	65.72	2,923.23	0.2317	677.31
20	48.93	65.72	3,215.68	0.2145	689.76
Total Present Value of all cost savings..........					$11,654.59

In comparing the results of Example I with Example H, one can see that the cost savings of the project are greater than its costs. Before the project is implemented, the costs and cost savings of alternatives should be investigated. Primarily, the alternatives in this instance are collector areas of different sizes. As can be recalled from studying the marginal analysis techniques presented in Module EE-01, "Fundamentals of Energy Cost Analysis," the marginal cost of adding a number of square feet of collector space should be compared to the marginal cost savings, and the "best" collector area would be where the marginal cost equals the marginal cost savings. The process could go the other way: the cost savings associated with a smaller collector area — in the form of reduced installation cost — might be greater than the value of the lost energy. In this case, the number of square feet should be decreased until marginal cost equals marginal cost savings.

This module has included important calculations of the costs and cost savings of energy projects. When these calculations are accurately performed, it is more likely that proper decisions concerning the energy projects will be made.

EXERCISES

1. Computation of Payment Schedule.

A general contractor is building a new 80-home subdivision for which 80 new central air-conditioning (heating and cooling) systems must be purchased. If purchased from Company A, the units would cost $87,000. Company A is willing to provide the units for 10% down and a payment schedule as follows (based on 15% interest rate):

Year	Payment Due Dec. 31
1	$23,000.00
2	23,000.00
3	23,000.00
4	23,000.00
5	23,000.00
6	2,776.66

Company B will provide similar units for nothing down and $26,000 per year for 5 years.

a. Complete the following payment schedules, discounting at 10%:

COMPANY A					
Year	Payment	Interest Cost	Amt. Applied to Principal	Balance	Present Value of Payment
0	$ 8,700.00				
1	23,000.00				
2	23,000.00				
3	23,000.00				
4	23,000.00				
5	23,000.00				
6	2,776.66				
Total PV of cost from A........ =					

COMPANY B		
Year	Payment	PV of Payments
1	$26,000.00	
2	26,000.00	
3	26,000.00	
4	26,000.00	
5	26,000.00	
Total PV of cost from B .... =		

b. From which company should the units be bought?

c. If the contractor is in the 46% tax bracket, calculate the following additional columns of the contractor's schedule:

Year	Interest Cost After Tax	Amt. Applied to Principal (same as above)	Total Cost of Payments	PV of Payments
1				
2				
3				
4				
5				
6				

d. Tax have what effect on the total costs of a company?

2. Tax Rate and Tax Credits.

A major farm machinery company has determined that it is losing 80.93 billion Btu/year through chimney stacks in its production plant, as shown in Figure 4. Its engineering department knows that some of this heat could be saved by the installation of recuperators. The potential for heat recovery is shown as follows (Figure 4):

1. No 6 Belt Anneal — 83,287 Btu/hr — 5 burners.
   Recovery factor estimated at 30% of what is lost;
   83,287 Btu/hr x 24 hrs/day x 310 days/yr ÷ 1,000,000 x
   0.30 recovery factor x $2.35/MM Btu = $436.88/yr.

   Est Cost = $900 per burner x 5 burners = $4,500.

2. No 3 Bolt & Nut Furnace — 701,734 Btu/hr — 13 burners
   701,734 Btu/yr x 24 hrs x 310 days/yr ÷ 1,000,000 x
   0.30 factor x $2.35/MM Btu = $3,680.74/yr.

   Est Cost = $900 per burner x 13 burners = $11,700.

3. No 4 Bolt & Nut — Same as Item 2.

4. — 3 Lindberg Heavy Duty Pacemakers. 6,488,017 Btu/hr
   18 burners.

   6,488,017 Btu/hr x 24 hrs/day x 310 days/yr ÷ 1,000,000 x
   0.30 factor x $2.35/MM Btu = $34,030.95/yr.

   Est Cost = $900 per burner x 18 burners = $16,200.

5. No 14 Bearing Race Furnace — 6 burners.

   4,351 MM Btu/yr x 0.30 factor x $2.35/MM Btu
   = $3,067.46/yr.

   Est Cost = $900 per burner x 6 burners = $5,400.

6. No 3 Bearing Race Furnace — 6 burners.

   7,590 MM Btu/yr x 0.30 factor x $2.35/MM Btu
   = $5,350.95/yr.

   $900 per burner x 6 burners = $5,400.

7. No 2 Bearing Race Furnace — 6 burners.

   5,515 MM Btu/yr x 0.30 factor x $2.35/MM Btu
   = $3,888.08/yr.

   Est Cost = $900 per burner x 6 burners = $5,400.

8. 8 Row Pusher Furnace.

   4,232 MM/Btu/yr x 0.30 factor x $2.35/MM Btu
   = $2,983.56/yr.

   Est Cost = $900 per burner x 17 burners = $15,300.

Summary:

Energy Saved — 228.167 therms of gas/yr.

Dollars Saved — $53,619.34 per year.

Estimated Cost — $75,600.

Figure 4. Calculations Showing the Savings by Use of Recuperators on Furnaces.

Note: The estimated life of the recuperators is 10 years.

a. If the cost of the fuel escalates at 15%/year, what is the net present value of installing the recuperators (Discounting at 10%)? Net present value is defined as "the present value of benefits minus the present value of costs."

A	B	C	D	E
Year	Escalated Fuel Cost Savings at 15%/yr	PVIF	PV Fuel cost Savings	PV Cost of Recuperator
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Total PV of Fuel Cost Savings = ☐

Total PV of Recuperator Costs = ☐

CALCULATIONS:

Column B: Previous Year Cost x 1.15

Year 1 = $53,619.34 x 1.15 = $61,662.24

Year 2 = $61,662.24 x 1.15 = $70,911.58

etc.

Column C: From Data Table 2, 10% rate

Column D = B x C

Column E = $75,600 cost from Figure 4

b. The farm machinery company is in a 46% tax bracket and is eligible for a $20,000 investment tax credit. What is the after-tax credit? What is the after-tax net PV of installing the recuperators?

Total Cost Savings (after tax) =
TCS x (1 - 0.46) + $20,000 = ____________
Total Costs (after tax) =
TC x (1 - 0.46) = ____________
NPV (after tax) = TCS (AT) - TC (AT)
= ________ - _______ = _______.

DATA TABLE 1. FUTURE VALUE OF $1 (FVIF).

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1200	1.1400	1.1500	1.1600	1.1800	1.2000	1.2400	1.2800	1.3200	1.3600
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2544	1.2996	1.3225	1.3456	1.3924	1.4400	1.5376	1.6384	1.7424	1.8496
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.4049	1.4815	1.5209	1.5609	1.6430	1.7280	1.9066	2.0972	2.3000	2.5155
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5735	1.6890	1.7490	1.8106	1.9388	2.0736	2.3642	2.6844	3.0360	3.4210
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.7623	1.9254	2.0114	2.1003	2.2878	2.4883	2.9316	3.4360	4.0075	4.6526
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.9738	2.1950	2.3131	2.4364	2.6996	2.9860	3.6352	4.3980	5.2899	6.3275
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.2107	2.5023	2.6600	2.8262	3.1855	3.5832	4.5077	5.6295	6.9826	8.6054
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.4760	2.8526	3.0590	3.2784	3.7589	4.2998	5.5895	7.2058	9.2170	11.703
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.7731	3.2519	3.5179	3.8030	4.4355	5.1598	6.9310	9.2234	12.166	15.916
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	3.1058	3.7072	4.0456	4.4114	5.2338	6.1917	8.5944	11.806	16.059	21.646
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.4785	4.2262	4.6524	5.1173	6.1759	7.4301	10.657	15.111	21.198	29.439
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.8960	4.8179	5.3502	5.9360	7.2876	8.9161	13.214	19.342	27.982	40.037
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	4.3635	5.4924	6.1528	6.8858	8.5994	10.699	16.386	24.758	36.937	54.451
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.8871	6.2613	7.0757	7.9875	10.147	12.839	20.319	31.691	48.756	74.053
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	5.4736	7.1379	8.1371	9.2655	11.973	15.407	25.196	40.564	64.358	100.71
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	6.1304	8.1372	9.3576	10.748	14.129	18.488	31.242	51.923	84.953	136.96
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	6.8660	9.2765	10.761	12.467	16.672	22.186	38.740	66.461	112.13	186.27
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	7.6900	10.575	12.375	14.462	19.673	26.623	48.038	85.070	148.02	253.33
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	8.6128	12.055	14.231	16.776	23.214	31.948	59.567	108.89	195.39	344.53
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	9.6463	13.743	16.366	19.460	27.393	38.337	73.864	139.37	257.91	468.57
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	10.803	15.667	18.821	22.574	32.323	46.005	91.591	178.40	340.44	637.26
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	12.100	17.861	21.644	26.186	38.142	55.206	113.57	228.35	449.39	866.67
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	13.552	20.361	24.891	30.376	45.007	66.247	140.83	292.30	593.19	1178.6
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	15.178	23.212	28.625	35.236	53.108	79.496	174.63	374.14	783.02	1602.9
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.834	17.000	26.461	32.918	40.874	62.668	95.396	216.54	478.90	1033.5	2180.0
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.918	19.040	30.166	37.856	47.414	73.948	114.47	268.51	612.99	1364.3	2964.9
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.245	13.110	21.324	34.389	43.535	55.000	87.259	137.37	332.95	784.63	1800.9	4032.2
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.167	14.421	23.883	39.204	50.065	63.800	102.96	164.84	412.86	1004.3	2377.2	5483.8
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.172	15.863	26.749	44.693	57.575	74.008	121.50	197.81	511.95	1285.5	3137.9	7458.0
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.062	13.267	17.449	29.959	50.950	66.211	85.849	143.37	237.37	634.81	1645.5	4142.0	10143.
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.285	14.974	21.724	31.409	45.259	93.050	188.88	267.86	378.72	750.37	1469.7	5455.9	19426.	66520.	*
50	1.6446	2.6916	4.3839	7.1067	11.467	18.420	29.457	46.901	74.357	117.39	289.00	700.23	1083.6	1670.7	3927.3	9100.4	46890.	*	*	*
60	1.8167	3.2810	5.8916	10.519	18.679	32.987	57.946	101.25	176.03	304.48	897.59	2595.9	4383.9	7370.1	20555.	56347.	*	*	*	*

*FVIF > 99,999

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GLOSSARY

Construction costs: The costs of getting a workable project into operation.

Costing period: The period of time over which energy investment costs are calculated. It is determined by mangement.

Down payment: The initial amount paid out of internal funds for an energy project.

Engineering, design, and development costs: The costs of making a project workable.

Installments: A regular payment equal to the interest charge plus some amount to reduce the principal of a loan.

Investment: A expenditure designed to reduce costs or increase income.

Life-cycle costing: The division of costs into three categories: (1) engineering, design, and development; (2) construction; and (3) operating.

Life of an investment: The length of time a system will continue to function.

Marginal tax rate: The percentage tax on the last dollar of earned income.

Operating costs: The costs of keeping an energy system going.

Outstanding balance: The principal of a loan minus all payments made to reduce the principal.

Tax bracket: The most recent tax rate for a person or business.

Tax credit: An expenditure which directly decreases a corporation's tax bill.

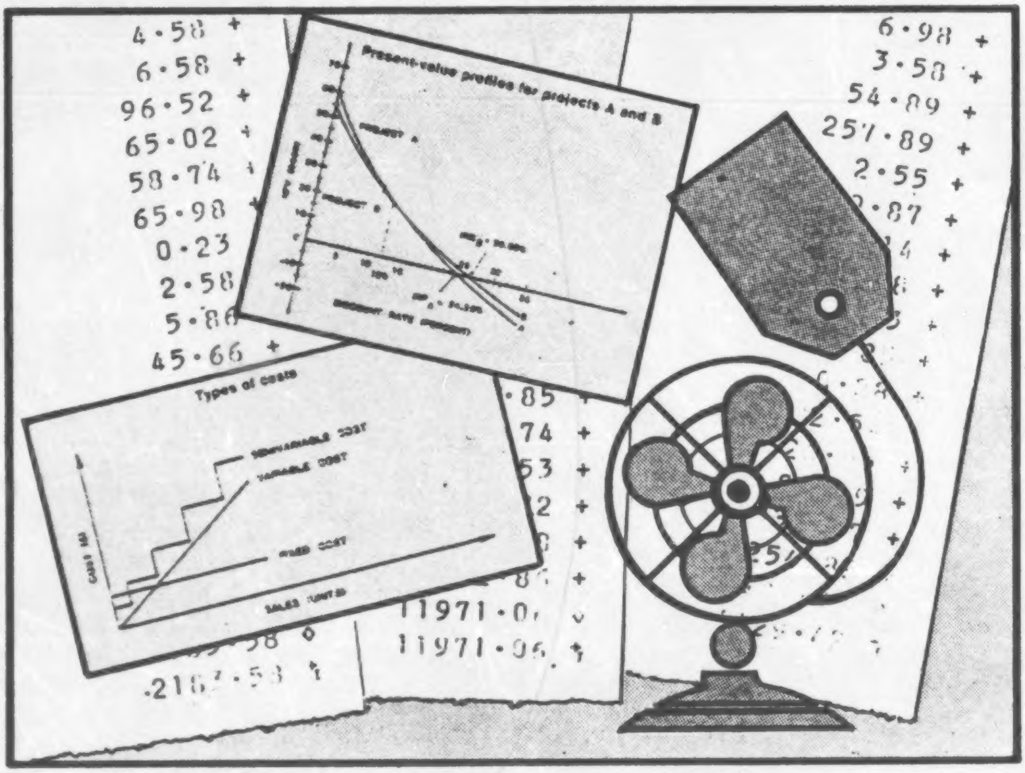
Tax deductible: Describes an expenditure which decreases a corporation's taxable income.

Tax rate: The amount of taxes that must be paid divided by the total income subject to taxes.

ENERGY TECHNOLOGY

CONSERVATION AND USE

ENERGY ECONOMICS



EE-05

ENERGY ANALYSIS

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INTRODUCTION

The goals of the energy specialist are to encourage maximum energy-use productivity, economic efficiency, and cost minimization. These goals are accomplished by proper application of certain economic principles and techniques in a number of different situations. In most instances, the principles and techniques of energy economics will be applied when some course of action related to energy production, conservation, and use is under consideration. A number of factors must be considered with regard to any particular energy project. Some of these factors are identical for all projects; some apply only to specific kinds of projects; and some apply only to unique situations.

This module presents several techniques that are used in the analysis of ways in which costs and cost savings of energy-conservation projects relate to one another. The calculations involved in these techniques are explained and demonstrated, and the information needed to perform each calculation and the results of each calculation are emphasized, as well as when each method should be used. With these techniques, the specialist will be able to analyze accurately the economic effects of energy-conservation projects or groups of projects.

PREREQUISITES

The student should have a good understanding of basic algebraic functions and should have completed Modules EE-01, EE-02, EE-03, and EE-04 of Energy Economics.

OBJECTIVES

Upon completion of this module, the student should be able to:

1. Define the following terms:
    a. Replacement project.
    b. Original project.
    c. Mutually-exclusive projects.
    d. Net cost savings.
    e. Payback period.
    f. Capital recovery factor.
    g. Benefits.
    h. Benefit-cost ratio.
    i. Net present value.
    j. Internal rate of return.
2. Distinguish between original projects and replacement projects and tell whether a group of projects is mutually-exclusive.
3. State what information is needed to calculate or determine the following:
    a. Capital recovery factor.
    b. Amount of cost savings needed per year to recover original cost.
    c. New cost savings.
    d. Net present value.
    e. Internal rate of return.
    f. Payback period.
4. State the nature of the information provided by the following methods:
    a. Payback period.
    b. Benefit-cost analysis.
    c. Net present value.
    d. Internal rate of return.

5. Given the proper information, calculate the following:
    a. Payback period.
    b. Benefit-cost ratio.
    c. Net present value.
    d. Internal rate of return.
6. Utilize the above methods when the cost savings per year form an irregular flow — that is, when they are not the same amount every year.
7. Examine a group of energy conservation projects and, through economic analysis, determine which should be implemented.

REPLACEMENT PROJECTS AND ORIGINAL PROJECTS

Energy projects can be placed in the following two categories: (1) replacement projects and (2) original projects. A replacement project is a course of action that changes or modifies an energy-related system already in use. An example of a replacement project is reducing the ventilation in a building by changing the air vents in each room. An original project involves the production and/or installation of an energy-related system where the function to be performed by the system is performed for the first time when the system is implemented. Two examples of original projects are the installation of a HVAC system in a new building and the purchase of a microcomputer to eliminate unnecessary energy use. The energy specialist must be able to examine a project and determine whether it is a replacement project or an original project.

With an original project, there are two factors to consider: (1) exactly what needs to be done, and (2) what different ways of doing it are available. If a new building is to be heated, for instance, there are a number of alternatives available. It is the function of the energy specialist to ensure that the new system is as economical as possible. With a replacement project, there also are two important considerations: (1) can the replacement project save money, and (2) what alternatives are there. Once again, the energy specialist must be able to decide which alternative is the most economical and still perform the desired function. The techniques presented in this module will help the energy specialist to evaluate alternative ways of implementing replacement projects and original projects.

MUTUALLY-EXCLUSIVE PROJECTS

Two or more energy conservation projects are mutually-exclusive if they perform the same function. For instance, a solar-heating system with 400 square feet of collector space and one with 600 square feet of collector space are mutually exclusive projects since they both can heat a home or building. Whenever two or more projects are mutually-exclusive, only one should be implemented. In a case where more than one of the projects are profitable, then the most profitable one should be chosen. In other words, the solar system which has cost savings that exceed its costs by the greatest amount should be implemented. If two or more projects are not mutually-exclusive, then each profitable project should be implemented.

NET COST SAVINGS PER YEAR

A concept that is sometimes useful in the analysis of energy projects is the concept of net cost savings per year. This technique considers each year separately and determines the costs and cost savings of that year. Then the following formula is used:

$$\begin{array}{c}\text{Net cost savings}\\ \text{for year } n\end{array} = \begin{array}{c}\text{Cost savings}\\ \text{in year } n\end{array} - \begin{array}{c}\text{Cost in}\\ \text{year } n\end{array} \qquad \text{Equation 1}$$

This technique is acceptable because the costs and cost savings occur in the same time period and are comparable. Once the net savings are found, they can be discounted back and compared to the original cost of the energy project.

EXAMPLE A: NET COST SAVINGS.	
Given:	A solar water-heating system costing $5000 will reduce electricity bills by $800 per year. The system will incur maintenance costs of $100 per year.
Find:	The net cost savings per year of the project.
Solution:	Use Equation 1: Net cost savings/yr = Cost savings/yr - costs/yr = $800 - $100 = $700/yr.

It should be noted that the costs and cost savings do not have to be the same each year to use this technique. If the costs and cost savings of any particular year are known, the net cost savings can be computed.

PAYBACK PERIOD

One common way to evaluate the costs and cost savings of an energy project is called the payback period. The payback period is defined as "the length of time it takes a business or an individual to obtain net cost savings equal to the cost of the investment." If the savings each year are constant, the following formula can be used:

$$\text{Payback period} = \frac{\text{Initial cost}}{\text{Net savings/yr}} \qquad \text{Equation 2}$$

EXAMPLE B: PAYBACK PERIOD WHEN SAVINGS EACH YEAR ARE EQUAL.

Given: An individual installs a microcomputer in his or her home to encourage energy conservation. The system has an original cost of \$4000 and will generate net after-tax savings of \$500 per year.

Find: The payback period of this investment.

Solution: Use Equation 2.

$$\begin{aligned}\text{Payback period} &= \frac{\text{Initial cost}}{\text{Net savings/yr}}\\ &= \frac{\$4000}{\$500/\text{yr}}\\ &= 8 \text{ years.}\end{aligned}$$

If the savings each year are not constant, then Equation 2 is not applicable. In this case, the savings from each year are added until their sum equals the original investment. The year when the cost savings equal the cost is the payback period. The following example should make this more clear.

EXAMPLE C: PAYBACK PERIOD WHEN SAVINGS EACH YEAR ARE NOT CONSTANT.

Given: In this example, suppose that net cost savings are $500 for the first year and will increase 12% each year thereafter, whereas the original price of the system remains at $4000.

Find: The payback period of this investment.

Solution: Consider the following table:

TABLE 1. PAYBACK PERIOD WITH AN IRREGULAR FLOW OF COST SAVINGS.

Year	Original savings (1st year)	Escalation FVIF 12%	Actual savings for year	Cumulative savings at end of year
0	$500	Given	$500.00	$ 500.00
1	500	1.1200	560.00	1060.00
2	500	1.2544	627.20	1687.20
3	500	1.4049	702.45	2389.65
4	500	1.5735	786.75	3176.40
5	500	1.7623	881.16	4057.56

As Table 1 shows, the total cost savings of the project become equal to its cost at the end of the fifth year. Thus, a rough estimate of the payback period is 5 years.

Once the payback period has been determined, it can be examined in light of the expected life of the investment and the amount of money being spent. If the payback period is a short period of time, the investment can be considered attractive.

The payback-period method of evaluating energy conservation projects has two very important drawbacks. The first is that the method does not take into account the time value of money. The net cost savings of the fifth year are counted equally with those of the first year. Doing this tends to overstate the actual amount of net cost savings. Second, the method ignores net cost savings beyond the payback period. An investment might be rejected as having a payback period that is too lengthy, although, in reality, it could have generated substantial savings after the payback period – which would have made it profitable. In general, the payback-period method should be used only as a quick, first method of looking at investment proposals. It should never be used as the only basis of accepting or rejecting an energy conservation project.

CAPITAL RECOVERY FACTOR

The capital recovery factor is a number that shows what percentage of the original cost of an investment must be generated each year in net cost savings for the original amount spent to be recovered. The term "capital" is used to describe the original amount of money spent on the energy conservation project. This technique is similar to the payback period method in that its emphasis is on recovering the original cost of the project. However, it does account for the time value of money and can be used in a technique known as benefit-cost analysis to determine the profitability of a proposed investment. This technique assumes that the net cost savings of each year will be identical. The following formula can be used to obtain the amount of cost savings needed per year to recover the original amount invested.

$$\text{Net cost savings needed/yr to recover original investment} = \text{Amount of original investment} \times \text{Capital recovery factor} \qquad \text{Equation 3}$$

To obtain the appropriate capital recovery factor, the following information is needed: the life of the investment and the applicable interest rate. This information can be used to obtain the capital recovery factor from a Capital Recovery Factor Table similar to the one shown in Table 2.

TABLE 2. CAPITAL RECOVERY FACTOR.

Years	Interest Rate (%)		
	10%	12%	15%
5	0.2638	0.2774	0.2983
8	0.1874	0.2013	0.2228
10	0.1628	0.1770	0.1992
15	0.1315	0.1468	0.1710
20	0.1175	0.1339	0.1598
25	0.1102	0.1275	0.1547
30	0.1061	0.1241	0.1523

The process of finding the net cost savings needed per year is demonstrated in Example D.

EXAMPLE D: USE OF THE CAPITAL RECOVERY FACTOR.

Given: An energy conservation project with an expected life of 20 years has an original cost of $3300. The interest rate is 10%.

Find: The net savings needed per year to recover the original $3300.

Solution: Table 2 shows that the capital recovery factor associated with 20 years and 10% is 0.1175. Therefore, Equation 3 can be used as follows:

$$\begin{array}{l} \text{Net cost savings} \\ \text{needed/yr to} \\ \text{recover original} \\ \text{investment} \end{array} = \begin{array}{l} \text{Amount of} \\ \text{original} \\ \text{investment} \end{array} \times \begin{array}{l} \text{Capital} \\ \text{recovery} \\ \text{factor} \end{array}$$

$$= \$3300 \times 0.1175$$

$$= \$387.75.$$

So, if the project generated net cost savings of $387.75/yr for 20 years, then the present value of the savings would equal $3300 if discounted at a rate of 10%.

BENEFIT-COST ANALYSIS

The net cost savings of an energy project are often called the benefits of the project. If the actual net cost savings per year are identical and known, they can be used to compute the benefit-cost ratio. The formula for this ratio is as follows:

$$\text{Benefit-cost ratio} = \frac{\text{Actual net cost savings/yr}}{\text{Net cost savings/yr needed to recover original investment}} \qquad \text{Equation 4}$$

The lower part of the formula shown in Equation 4 is obtained by using Equation 3. If the benefit-cost ratio is larger than one, then the net cost savings of a project (or its benefits) exceed its cost and the investment is profitable. On the other hand, if the number obtained by Equation 4 is less than one, then the net cost savings needed per year are greater than those actually obtained and the investment is not profitable.

EXAMPLE E: BENEFIT-COST RATIO.

Given: The assumption that the project in Example D results in net cost savings of $500 per year.

Find: The benefit-cost ratio and determine whether the project is profitable or not.

Solution: Example D shows the net cost savings needed to recover the original investment are $387.85. Therefore, Equation 4 can be used as follows:

$$\text{Benefit-cost ratio} = \frac{\text{Actual net cost savings/yr}}{\text{Net cost savings needed/yr to recover original investment}}$$

$$= \frac{\$500}{\$387.75}$$

$$= 1.2895.$$

Since the benefit-cost ratio is greater than one, the investment is profitable.

It should be noted that if the benefit-cost ratio is exactly equal to one, then the energy project will neither save money nor cost money; it will save exactly as much as it costs. The benefit-cost ratio is an excellent way to illustrate how the cost savings of an energy conservation project relates to its cost.

NET PRESENT VALUE METHOD

In Module EE-02, the concept of present value was presented. If a fixed amount of money or a series of equal, yearly cost savings (an annuity) is associated with a certain time period, then an amount associated with the present – which is equal to the future amount – can be calculated. The latter figure is the present value of the fixed amount, or the annuity. This process is called discounting. Because an amount of money in the present is worth more than the same amount at any point in the future (due to the time value of money), all dollar amounts associated with an energy project should be converted to the same time period. By using the techniques presented in Modules EE-02 and EE-03, present-value calculations can be used as part of an analysis that can help determine whether an energy project should be undertaken.

DEFINITION OF NET PRESENT VALUE

All energy projects involve costs and/or cost savings. In Module EE-02, the following principle was presented: If the present value of the cost savings of a project exceeds its cost, the project is profitable. In many cases the costs of a project do not occur only when the project is undertaken. In these instances, the present value of the costs can be calculated and compared to the present value of the cost savings — as was done in Module EE-04 with life-cycle costing. If the present value of the savings exceeds the present value of the costs, the project is profitable.

Another way to view this process is as follows: If the present value of the cost savings is greater than the present value of the costs, then, when the present value of the costs is subtracted from the present value of the savings, the result should be a positive number. This difference is defined as "net present value."

$$\text{NPV} = \text{PV (cost savings)} - \text{PV (costs)} \qquad \text{Equation 5}$$

where:

NPV = Net present value.

PV (Costs) = Present value of amount in parenthesis.

Equation 5 leads to the following rule:

NPV Rule 1: For a given energy project, if the net present value of that project is greater than zero, the project is profitable.

CALCULATION OF NET PRESENT VALUE

The calculation of net present value is a simple extension of present-value calculation techniques presented in Modules EE-02 and EE-03. The first step is to identify the costs and cost savings in each subsequent year. As noted before, this process involves the consideration of several factors. Once these costs and cost savings have been determined and associated with some time period, each amount should be discounted back to the present in the manner outlined in Module EE-02 (that is, their present values should be obtained). If some of these figures form annuities, then annuity techniques should be used. The present values associated with the costs of each year should be summed to obtain the present value of the total cost of the project. Similarly, the present values associated with the cost savings of each year should be summed to get the present value of the total cost savings of the project. Then these values can be used in Equation 5 to determine the net present value. Example F illustrates this process:

EXAMPLE F: NET PRESENT VALUE.	
Given:	In a cement plant, three grinding equipment gear trains are sprayed manually with gear compound every 2 hours, using 4350 gal lubricant/yr. An automatic oil-mist system uses only 150 gal/yr, while still lubricating adequately. The gear lubricant costs $0.2725/lb. The expected life of the system is 5 years. The system would cost $7500 to purchase and install. Maintenance costs would be $450/yr. The interest rate is 9%.

Example F. Continued.

Find: The net present value of installing the oil-mist system.

Solution: Gallons saved yr = 4350 gal - 150 gal
= 4200 gal.

Now, since there are 8 lb lubricant/gal,

Cost savings yr = 4200 gal x 8 lb/gal x \$0.2725/lb
= \$9156/yr.

Thus, the cost savings associated with this project form an annuity of \$9156/yr for 5 years. The present value of this annuity is given as follows:

PV(cost savings) = \$9156 x $PVIF_a$ (5 years, 9%)
= \$9156 x 3.889
- \$35,607.68.

The present value of the costs of the project is computed as follows:

PV(maintenance costs) = \$450 x $PVIF_a$ (5 years, 9%)
= \$450 x 3.889
= \$1750.05

Now, the present value of the purchase and installation cost is just \$7500 since it is done in the present:

PV (all costs) = \$7500 + \$1750.05
= \$9250.05

Now, using Equation 5, the following NPV is obtained:

NPV = PV (cost savings) - PV (all costs)
= \$35,607.68 - \$9250.05
= \$26,357.63

Example F. Continued.

> Since the net present value is greater than zero (in fact, by a very substantial amount), the project should be implemented.*

Example F also can be calculated by using the concept of net cost savings, as shown previously in Equation 1:

$$
\begin{aligned}
\text{Net cost savings/yr} &= \text{Cost savings/yr} - \text{Cost/yr} \\
&= \$9156 - \$450 \\
&= \$8706/\text{yr.}
\end{aligned}
$$

$$
\begin{aligned}
\begin{array}{l}\text{Present value of}\\ \text{net cost savings}\end{array} &= \text{Net savings/yr} \times \text{PVIF}_a\ (5\ \text{years},\ 9\%) \\
&= \$8706 \times 3.889 \\
&= \$33{,}857.63
\end{aligned}
$$

$$
\begin{aligned}
\text{Net present value} &= \begin{array}{l}\text{Present value of}\\ \text{net cost savings}\end{array} - \text{Original cost} \\
&= \$33{,}857.63 - \$7500 \\
&= \$26{,}357.63.
\end{aligned}
$$

USEFULNESS OF THE NET PRESENT VALUE (NPV) METHOD

At this point the student may not feel that the concept of net present value is any more useful than the concepts presented in Module EE-02. Indeed, when analyzing one particular project, the NPV method offers no special advantage. However, the NPV method does become most useful when a particular function can be performed in more than one way (that is, by more than one system). In this instance, the project with the highest NPV should be chosen. As discussed previously, when two or more alternative projects perform the same function, the projects are mutually-exclusive. And, if the NPV of each project in a group of mutually-exclusive projects can be calculated, the one with the highest NPV is the one that should be implemented. In this case, the project with the higher NPV has either greater cost savings or smaller costs — or both.

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*This example is taken in part from NBS Handbook 115, Supplement 1, page 3-59.

Suppose that in Example F four different oil-mist systems were available at differing costs. Each system could reduce costs by a different amount, and each had a NPV greater than zero (total cost savings exceed total costs in present-value terms).

TABLE 3. CHOICES AMONG ALTERNATIVE OIL-MIST SYSTEMS.

System	Purchase Price	Maintenance Cost	Cost Savings	NPV
1	$3000	$550	$4000	$10,417.05
2	$5000	$500	$6000	$16,389.50
3	$7500	$450	$9165	$26,357.63
4	$9000	$200	$9300	$26,389.90

In this example, System 4 would be implemented since it has the highest NPV. Once the alternative ways of achieving a goal have been determined, the NPV method can be used to choose from among the alternatives.

At this point, a few words should be mentioned concerning the cases where there is a group of energy projects that are not mutually-exclusive. Since each project will perform a different function, each can be considered individually. Thus, each project with an NPV greater than zero should be implemented. This principle, together with the previously stated one, is expressed as follows:

> NPV Rule 2: From a group of mutually-exclusive projects, only the one with the highest NPV should be implemented (if that NPV is greater than zero). From a group of projects that are not mutually-exclusive, each project with an NPV greater than zero should be implemented.

A group of possible projects that is not mutually-exclusive is likely to be the result of an intense energy audit.

It should be noted that the net present value of an energy project with an irregular flow of net cost savings can be calculated by simply taking the present value of the net cost savings each year, summing them, and then subtracting the original cost of the project.

SHORTCOMINGS OF THE NPV METHOD

Generally, the NPV method is an effective way of reaching decisions or obtaining information about prospective energy projects. The NPV method does have one flaw, however: Whenever the present value is calculated (and, therefore, whenever NPV is calculated), an interest rate must be used. Unfortunately, more than one interest rate exists in the economy at any given time. In addition, the applicable interest rate can be different for different organizations and can change from time to time for the same organization. Oftentimes, the determination of the actual rate to use in the discounting process is a long and complex task beyond the concern of the energy specialist. In actual practice, the interest rate to use in calculations of the nature described in this module often can be obtained from management personnel. Even so, in many cases it is useful to have some measure of the nature of an energy project that does not depend on interest rate. In the next section of this module, a measure of this kind will be developed.

INTERNAL RATE OF RETURN (IRR) METHOD

When the internal rate-of-return (IRR) method is used to evaluate an energy project, the applicable interest rate does not need to be known. The known data regarding costs and cost savings are used to calculate a percentage figure (rate) that indicates some information about the monetary effect of implementing a certain energy project.

DEFINITION OF INTERNAL RATE OF RETURN

All energy projects have associated costs and cost savings. (For a project with no cost – such as turning down the thermostat at night in an office building during the heat season – it is said that cost = $0.) The IRR method determines the interest rate – which, if used to discount the net cost savings per year back to the present, would yield a present value of total net cost savings exactly equal to the initial cost of the project. (Whereas the word "exactly" is used in the definition, in actual practice it is usually "acceptable" and often "necessary" to approximate.) In other words, the IRR of a point is the particular interest rate — which, if used to discount the net cost savings, would result in a net present value equal to zero. This rate of return reflects the extent of the net cost savings of the project as compared to its initial cost. Just as a bank account brings a specified return on the initial amount deposited, so will an energy project generate some return on its initial cost. The higher the internal rate of return of a project, the greater are the associated cost savings relative to the project's initial cost. Because of this fact, the following observation can be made:

> IRR Rule 1: If an interest rate used to discount a series of net cost savings is less than the internal rate of return of a project, then the net present value of that project will be greater than zero. If the interest rate used is greater than the IRR, then the NPV of the project will be less than zero. If the interest rate used is equal to the IRR, then (by definition) the NPV is equal to zero.

IRR Rule 1 is easy to understand if the observation about discounting made in Module EE-02 is understood. The observation was that the higher the interest rate used to discount a fixed amount or annuity, the smaller is its present value. In this case, the search is being made for the one particular interest rate that will, when used to discount the cost savings, result in a present-value figure equal to the initial cost of the project (which implies NPV = 0). Once that interest rate is found, increasing the discount rate makes the present value of cost savings smaller and, thus, decreases NPV. Since NPV = 0 when a discount rate equal to the IRR is used, a higher discount rate results in a net present-value figure less than zero. On the other hand, a smaller interest rate will lead to a higher present value of cost savings – with the result being a higher net present value. Since NPV = 0 when a discount rate equal to the IRR is used, a lower discount rate results in a net present-value figure greater than zero. These observations are outlined in Table 4.

TABLE 4. EFFECT OF DISCOUNT RATE ON NPV.

Given: An energy project with a specified internal rate of return (IRR).

Interest Rate (r) Used to Discount Cost Savings	Magnitude of NPV
$r < IRR$	$NPV > 0$
$r = IRR$	$NPV = 0$ (by definition)
$r > IRR$	$NPV < 0$

The facts presented in Table 4 have an important implication. If the applicable rate of interest (which would be used to discount cost savings) is less than the internal rate of return of an energy project, then the NPV of the project is greater than zero. Therefore, the project should be implemented. Even though the relevant interest rate is difficult to determine, management will often have some idea of the relevant range of interest rates. For example, suppose the relevant interest rate is known to be somewhere between 8% and 14%. In this instance, a project with a 15% or greater internal rate of return should be implemented, regardless of the relevant interest rate. As long as it is chosen in the given range, the NPV of the project will be greater than zero. If the IRR of the project falls between 8% and 14%, then management will have to consider other factors in deciding whether to implement the project. If the IRR is less than 8%, the project will never be undertaken.

Most energy project implementation decisions will be made by management and not by energy specialists. Because of this fact, it is often very useful for the specialist to be able to determine the IRR of a certain project and then let management decide what interest rate to use. The primary concerns of the energy specialist are the costs and cost savings of a proposed energy conservation project.

CALCULATION OF THE INTERNAL RATE OF RETURN (IRR)

The internal rate of return can be calculated easily with the use of the Present Value of an Annuity of $1 Table (Data Table 4). The present value of the net cost savings of an energy project is calculated by using this table when the savings are the same for each year; otherwise the savings would not form an annuity. The IRR is the interest rate which, if used to discount the annuity formed by the net cost savings, would result in a present-value figure approximately equal to the initial cost of the project. The following should be remembered:

$$\begin{array}{c}\text{Present value of}\\ \text{net cost savings}\end{array} = \begin{array}{c}\text{Net cost}\\ \text{savings/yr}\end{array} \times \text{PVIF}_a$$

Equation 6

Since the IRR results in PV of cost savings equal to initial cost, Equation 6 can be rewritten as follows:

$$\begin{array}{c}\text{Initial cost}\\ \text{of project}\end{array} = \begin{array}{c}\text{Net cost}\\ \text{savings/yr}\end{array} \times \begin{array}{c}\text{Needed}\\ \text{PVIF}_a\end{array}$$

Equation 7

This entry in Data Table 4 will correspond to the internal rate of return. Notice that Equation 7 can be modified to read as follows:

$$\begin{array}{c}\text{Needed}\\ \text{PVIF}_a\end{array} = \begin{array}{c}\text{Initial cost}\\ \text{of project}\end{array} \div \begin{array}{c}\text{Net cost}\\ \text{savings/yr}\end{array}$$

Equation 8

Using Equation 8, determine the $PVIF_a$ that is needed to result in PV of cost savings equal to initial cost. Then locate the number in Data Table 4 (in the appropriate time period row) closest to the number found in Equation 8. The interest rate associated with this number is the approximate internal rate of return. The following example should make this concept more clear.

EXAMPLE G: CALCULATE THE INTERNAL RATE OF RETURN (IRR).

Given: The manager of a warehouse is considering the installation of an air lock at the loading door. The size of the door, which is used for railroad cars, is 20 ft x 17.5 ft. The door is open for 10 minutes – 12 times per day – 5 days per week. The inside building temperature is 70°F. The heating season is October – April (30 weeks). The average outside temperature during the heating season is 38.4°F. The air flow velocity through the open door is 500 fpm. Steam, which supplies 960 Btu/lb, is used for heating. The cost of steam is $1.86/1000 lbs. Conversion factor: $0.0183 \text{ Btu} = 1 \text{ ft}^3 - {}^\circ\text{F}$. The cost of the air lock is $20,000 installed. Assume the life of the air lock is 30 years.

Find: The internal rate of return (IRR) of the purchase and installation cost of the air lock.

Example G. Continued.

Solution: Air entering door = 500 fpm x 20 ft x 17.5 ft

= 175,000 ft^3/min.

Temperature difference between inside and outside air = 70°F - 38.4°F = 31.6°F.

Heat loss = 175,000 ft^3 x 1 min x 0.0183 Btu/ft^3 - °F x 31.6°F

= 101,199 Btu/min.

Heating cost/min of door opening = 101,199 Btu/min x 1 lb steam/960 Btu x $1.86/1000 lb steam

= $0.196/min.

Annual cost savings = $0.196/min x 12 openings/day x 10 min/opening x 5 days/week x 30 weeks/yr

= $3528/yr.

Thus, the cost savings of this project for a 30-year annuity is $3528 per year. Now the process described in the text will be used to find the internal rate of return. The initial cost of the project is $20,000, and the net cost savings per year are $3528. Therefore, using Equation 8, the following is calculated:

Example G. Continued.

Proper PVIF_a (30 years, ?%) = \$20,000 ÷ \$3528 = 5.6689.

The Data Table 4, observe that the entries in the 30-year row are 25.808, 22.397, 19.600, and so on. The entry under the 16% column is 6.1772, and the entry under the 18% column is 5.5168. Since the entry needed to make the present value of the cost savings equal to the initial investment is 5.6689 – which is approximately one-fourth the difference between 5.5168 and 6.1772 – it can be concluded that the internal rate of return is approximately 17.5%.

In the case of Example G, if the relevant interest rate is less than approximately 17.5%, the project should be implemented. As stressed before, however, the main concern of the energy specialist should be in the determination of costs and cost savings and their uses to calculate NPV and IRR. This type of data supplied by the energy specialist will enable management to make more effective decisions, and an awareness of the information provided by this kind of analysis is essential to efficient energy use and cost minimization.

IRR AND AN IRREGULAR FLOW OF COST SAVINGS

Whenever the cost savings of an energy project are not the same each year, then the preceding techniques cannot be used to determine IRR. In this special case, a trial-and-error method must be used. The present value of the net cost savings of each year should be discounted back to the present and summed, using various interest rates. That interest rate, which leads to a total present value of net cost savings that is very close to the original cost of the project, is the approximate internal rate of return. A good interest rate to start with is 10%. If the total present value of net cost savings is greater than the original cost of the project, then a higher discount rate should be tried. Conversely, if the total present value of net cost savings is lower than the original cost, then a higher interest rate should be tried. This process can be time-consuming; however, it should take no more than three or four attempts to find the correct interest rate.

USE OF IRR AND EVALUATION OF MUTUALLY-EXCLUSIVE PROJECTS

As mentioned previously, when two or more projects that perform the same function can be implemented, the one with the higher NPV should be chosen. Another way to determine which project to implement is to calculate the IRR of each project. After this is done, the project with the highest IRR should be chosen – provided the IRR is high enough to justify the expenditure. This determination is expressed as follows:

IRR Rule: Given a group of mutually-exclusive projects, the one with the highest IRR should be implemented – provided that the IRR is high enough to justify the cost of the project.

COMPARISON OF METHODS FOR EVALUATING ENERGY PROJECTS

Each of the four methods for evaluating energy projects – capital recovery factor and benefit-cost analysis, net present value, internal rate of return, and payback period – provides unique information about the project under consideration. A comparison of the techniques is presented in Table 5.

TABLE 5. COMPARISON OF PROJECT EVALUATION METHODS.

Method	Needed Information	Result
Capital recovery factor and benefit-cost	Original cost; life of investment; relevant interect rate.	Needed cost savings per year to recover original investment; ratio that shows how cost savings relate to costs.
Net present value	Original cost; life of investment; relevant interest rate; net cost savings per year.	How much total present value of net cost savings exceeds (or is less than) original cost.
Internal rate of return	Original cost; life of net cost savings per year.	Interest rate which shows how net cost savings relate to original cost.
Payback period	Original cost; net cost savings per year.	Rough estimate of time needed to recover original cost.

The particular technique used for a given energy conservation project will depend on what information is available and what end result is required. Often, more than one technique will be used to provide maximum information. With the techniques presented in the five modules of this course, the specialist will be able to properly evaluate the economic effects of any energy conservation project.

EXERCISES

1. An energy specialist is examining the possible cost savings associated with the installation of on-line microcomputer controls to regulate the combustion processes in two steel treatment furnaces operating in parallel. The relevant parameters are as follows:

   - Each furnace has a throughput of 160 tons/day.
   - Computer control will result in fuel savings of 7%.
   - The furnaces require 1000 Btu of fuel per pound of steel treated.
   - The furnaces are powered by fuel oil that has an energy content of 138,500 Btu/gal when burned.
   - Computers of the type that would be installed have an average life of 8 years.
   - The cost of purchasing and installing the computer system would be $148,400.
   - Assume the price of fuel oil is $20/barrel.
   - The furnaces operate 350 days/yr.
   - The annual operating costs would be $17,146.

a. Find the internal rate of return. (Consider one furnace.)

Furnace throughput of steel/yr = 160 tons/day x 2000 lb/ton x 350 days/yr

= 112,000,000 lb/yr.

Energy use of furnace = 112,000,000 lb/yr x 1000 Btu/lb

= _______________ Btu/yr.

Energy savings/yr = _______________ Btu/yr x 0.07 savings use

= _______________ Btu/yr savings.

Fuel oil saved/yr = ______________ Btu/yr ÷ 138,000 Btu/gal fuel oil

= ______________ gals/yr.

Barrels saved/yr = ______________ gals/yr ÷ 42 gal/barrel

= ______________ barrels/yr.

Cost savings/yr/furnace = ______________ barrels/yr x $20/barrel

= ______________ $/yr.

Cost savings/yr for two furnaces = ______________ $/yr/furnace x 2 furnaces

= ______________ $/yr.

Net cost savings/yr = ______________ $/yr - $17,146 operating cost/yr

= ______________ $/yr.

Thus, the cost savings of installing the microcomputer form an annuity of $ _________ per year for 8 years. The internal rate of return is the interest rate which, if used to discount this annuity, would result in a present value of cost savings approximately equal to $148,400. First, find the entry from Data Table 4 (the 8-year column) which would lead to the following equation:

$48,400 = $ _________ Net cost savings/yr x Needed entry from Data Table 4 (8 years)

So,

Needed entry from Data Table 4 (8 years) = $48,400 ÷ $______ Net cost savings/yr

The interest rate associated with this energy is _____%; therefore, this is the IRR.

b. Find the net present value when the interest rate is 8%.

Present value of net cost savings = $________ x Proper entry from Data Table 4 (8 years, 8%)

= $________ x 5.747

= $________.

Net present value = PV of net cost savings - Initial cost

= $___________ - $148,400

= $___________.

c. Find the payback period when the interest rate is 8%. Find the entry in Data Table 4 (in the 8% column) which makes the PV of net cost savings equal to $148,400.

$48,400 = $_____ Net Cost savings/yr x Proper entry from Data Table 4 (8%, ? years)

So,

Proper entry from Data Table 4 (8%) = $48,400 ÷ $_________ Net cost savings/yr

The time period (n) which has a corresponding entry closest to this number is __________. This is the payback period in years.

d. Find the capital recovery factor.

Net cost savings/yr needed to recoup original investment = Amount of original investment x Capital recovery factor

= \$48,400 x 0.1490

Net cost savings/yr needed = \$____________.

Do the actual net cost savings per year exceed those needed as calculated by the capital recovery factor? What conclusions can be drawn from the capital recovery factor analysis?

2. An energy conservation project with an initial cost of \$1000 will result in cost savings of \$400 per year for 4 years. Find the following:

a. Find the internal rate of return:

Needed entry from Data Table 4 (given 4 years) = ________ ÷ ________

= ________.

The entry rate in the 4-year row which comes closest to this entry is ___________. The interst rate associated with this entry is ___________. This is is the IRR.

b. If the interest rate is 10%, find the net present value.

Present value of net cost savings = Net cost savings/yr x Entry from Data Table 4 (4 years, 10%)

= \$_________ x _________

= \$_________.

NPV = PV of cost savings - PV of costs

= $________ - $________

= $________.

c. If the interest rate is 10%, find the payback period. Recall Equation 7:

Needed entry from Data Table 4 (given 10%) = Initial cost ÷ Net cost savings/yr

= $________ ÷ ________

= ________.

The entry in the 10% column which comes closest to this number is ________, and its associated time period is ________. This is the approximate payback period.

DATA TABLE 1. FUTURE VALUE OF $1 (FVIF).

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000	1.1200	1.1400	1.1500	1.1600	1.1800	1.2000	1.2400	1.2800	1.3200	1.3600
2	1.0201	1.0404	1.0609	1.0816	1.1025	1.1236	1.1449	1.1664	1.1881	1.2100	1.2544	1.2996	1.3225	1.3456	1.3924	1.4400	1.5376	1.6384	1.7424	1.8496
3	1.0303	1.0612	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.4049	1.4815	1.5209	1.5609	1.6430	1.7280	1.9066	2.0972	2.3000	2.5155
4	1.0406	1.0824	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5735	1.6890	1.7490	1.8106	1.9388	2.0736	2.3642	2.6844	3.0360	3.4210
5	1.0510	1.1041	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.7623	1.9254	2.0114	2.1003	2.2878	2.4883	2.9316	3.4360	4.0075	4.6526
6	1.0615	1.1262	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.9738	2.1950	2.3131	2.4364	2.6996	2.9860	3.6352	4.3980	5.2899	6.3275
7	1.0721	1.1487	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.2107	2.5023	2.6600	2.8262	3.1855	3.5832	4.5077	5.6295	6.9826	8.6054
8	1.0829	1.1717	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.4760	2.8526	3.0590	3.2784	3.7589	4.2998	5.5895	7.2058	9.2170	11.703
9	1.0937	1.1951	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.7731	3.2519	3.5179	3.8030	4.4355	5.1598	6.9310	9.2234	12.166	15.916
10	1.1046	1.2190	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	3.1058	3.7072	4.0456	4.4114	5.2338	6.1917	8.5944	11.805	16.059	21.646
11	1.1157	1.2434	1.3842	1.5395	1.7103	1.8983	2.1049	2.3316	2.5804	2.8531	3.4785	4.2262	4.6524	5.1173	6.1759	7.4301	10.657	15.111	21.198	29.439
12	1.1268	1.2682	1.4258	1.6010	1.7959	2.0122	2.2522	2.5182	2.8127	3.1384	3.8960	4.8179	5.3502	5.9360	7.2876	8.9161	13.214	19.342	27.982	40.037
13	1.1381	1.2936	1.4685	1.6651	1.8856	2.1329	2.4098	2.7196	3.0658	3.4523	4.3635	5.4924	6.1528	6.8858	8.5994	10.699	16.386	24.758	36.937	54.451
14	1.1495	1.3195	1.5126	1.7317	1.9799	2.2609	2.5785	2.9372	3.3417	3.7975	4.8871	6.2613	7.0757	7.9875	10.147	12.839	20.319	31.691	48.756	74.053
15	1.1610	1.3459	1.5580	1.8009	2.0789	2.3966	2.7590	3.1722	3.6425	4.1772	5.4736	7.1379	8.1371	9.2655	11.973	15.407	25.196	40.564	64.358	100.71
16	1.1726	1.3728	1.6047	1.8730	2.1829	2.5404	2.9522	3.4259	3.9703	4.5950	6.1304	8.1372	9.3576	10.748	14.129	18.488	31.242	51.923	84.953	136.96
17	1.1843	1.4002	1.6528	1.9479	2.2920	2.6928	3.1588	3.7000	4.3276	5.0545	6.8660	9.2765	10.761	12.467	16.672	22.186	38.740	66.461	112.13	186.27
18	1.1961	1.4282	1.7024	2.0258	2.4066	2.8543	3.3799	3.9960	4.7171	5.5599	7.6900	10.575	12.375	14.462	19.673	26.623	48.038	85.070	148.02	253.33
19	1.2081	1.4568	1.7535	2.1068	2.5270	3.0256	3.6165	4.3157	5.1417	6.1159	8.6128	12.055	14.231	16.776	23.214	31.948	59.567	108.89	195.39	344.53
20	1.2202	1.4859	1.8061	2.1911	2.6533	3.2071	3.8697	4.6610	5.6044	6.7275	9.6463	13.743	16.366	19.460	27.393	38.337	73.864	139.37	257.91	468.57
21	1.2324	1.5157	1.8603	2.2788	2.7860	3.3996	4.1406	5.0338	6.1088	7.4002	10.803	15.667	18.821	22.574	32.323	46.005	91.591	178.40	340.44	637.26
22	1.2447	1.5460	1.9161	2.3699	2.9253	3.6035	4.4304	5.4365	6.6586	8.1403	12.100	17.861	21.644	26.186	38.142	55.206	113.57	228.35	449.39	866.67
23	1.2572	1.5769	1.9736	2.4647	3.0715	3.8197	4.7405	5.8715	7.2579	8.9543	13.552	20.361	24.891	30.376	45.007	66.247	140.83	292.30	593.19	1178.6
24	1.2697	1.6084	2.0328	2.5633	3.2251	4.0489	5.0724	6.3412	7.9111	9.8497	15.178	23.212	28.625	35.236	53.108	79.496	174.63	374.14	783.02	1602.9
25	1.2824	1.6406	2.0938	2.6658	3.3864	4.2919	5.4274	6.8485	8.6231	10.834	17.000	26.461	32.918	40.874	62.668	95.396	216.54	478.90	1033.5	2180.0
26	1.2953	1.6734	2.1566	2.7725	3.5557	4.5494	5.8074	7.3964	9.3992	11.918	19.040	30.166	37.856	47.414	73.948	114.47	268.51	612.99	1364.3	2964.9
27	1.3082	1.7069	2.2213	2.8834	3.7335	4.8223	6.2139	7.9881	10.245	13.110	21.324	34.389	43.535	55.000	87.259	137.37	332.95	784.63	1800.9	4032.2
28	1.3213	1.7410	2.2879	2.9987	3.9201	5.1117	6.6488	8.6271	11.167	14.421	23.883	39.204	50.065	63.800	102.96	164.84	412.86	1004.3	2377.2	5483.8
29	1.3345	1.7758	2.3566	3.1187	4.1161	5.4184	7.1143	9.3173	12.172	15.863	26.749	44.693	57.575	74.008	121.50	197.81	511.95	1285.5	3137.9	7458.0
30	1.3478	1.8114	2.4273	3.2434	4.3219	5.7435	7.6123	10.062	13.267	17.449	29.959	50.950	66.211	85.849	143.37	237.37	634.81	1645.5	4142.0	10143
40	1.4889	2.2080	3.2620	4.8010	7.0400	10.285	14.974	21.724	31.409	45.259	93.050	188.88	267.86	378.72	750.37	1469.7	5455.9	19426	66520	*
50	1.6446	2.6916	4.3839	7.1067	11.467	18.420	29.457	46.901	74.357	117.39	289.00	700.23	1083.6	1670.7	3927.3	9100.4	46890	*	*	*
60	1.8167	3.2810	5.8916	10.519	18.679	32.987	57.946	101.25	176.03	304.48	897.59	2595.9	4383.9	7370.1	20555	56347	*	*	*	*

*FVIF > 99,999

DATA TABLE 2. PRESENT VALUE OF $1 (PVIF).

Period	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174	.9091	.8929	.8772	.8696	.8621	.8475	.8333	.8065	.7813	.7576	.7353
2	.9803	.9612	.9426	.9246	.9070	.8900	.8734	.8573	.8417	.8264	.7972	.7695	.7561	.7432	.7182	.6944	.6504	.6104	.5739	.5407
3	.9706	.9423	.9151	.8890	.8638	.8396	.8163	.7938	.7722	.7513	.7118	.6750	.6575	.6407	.6086	.5787	.5245	.4768	.4348	.3975
4	.9610	.9238	.8885	.8548	.8227	.7921	.7629	.7350	.7084	.6830	.6355	.5921	.5718	.5523	.5158	.4823	.4230	.3725	.3294	.2923
5	.9515	.9057	.8626	.8219	.7835	.7473	.7130	.6806	.6499	.6209	.5674	.5194	.4972	.4761	.4371	.4019	.3411	.2910	.2495	.2149
6	.9420	.8880	.8375	.7903	.7462	.7050	.6663	.6302	.5963	.5645	.5066	.4556	.4323	.4104	.3704	.3349	.2751	.2274	.1890	.1580
7	.9327	.8706	.8131	.7599	.7107	.6651	.6227	.5835	.5470	.5132	.4523	.3996	.3759	.3538	.3139	.2791	.2218	.1776	.1432	.1162
8	.9235	.8535	.7894	.7307	.6768	.6274	.5820	.5403	.5019	.4665	.4039	.3506	.3269	.3050	.2660	.2326	.1789	.1388	.1085	.0854
9	.9143	.8368	.7664	.7026	.6446	.5919	.5439	.5002	.4604	.4241	.3606	.3075	.2843	.2630	.2255	.1938	.1443	.1084	.0822	.0628
10	.9053	.8203	.7441	.6756	.6139	.5584	.5083	.4632	.4224	.3855	.3220	.2697	.2472	.2267	.1911	.1615	.1164	.0847	.0623	.0462
11	.8963	.8043	.7224	.6496	.5847	.5268	.4751	.4289	.3875	.3505	.2875	.2366	.2149	.1954	.1619	.1346	.0938	.0662	.0472	.0340
12	.8874	.7885	.7014	.6246	.5568	.4970	.4440	.3971	.3555	.3186	.2567	.2076	.1869	.1685	.1372	.1122	.0757	.0517	.0357	.0250
13	.8787	.7730	.6810	.6006	.5303	.4688	.4150	.3677	.3262	.2897	.2292	.1821	.1625	.1452	.1163	.0935	.0610	.0404	.0271	.0184
14	.8700	.7579	.6611	.5775	.5051	.4423	.3878	.3405	.2992	.2633	.2046	.1597	.1413	.1252	.0985	.0779	.0492	.0316	.0205	.0135
15	.8613	.7430	.6419	.5553	.4810	.4173	.3624	.3152	.2745	.2394	.1827	.1401	.1229	.1079	.0835	.0649	.0397	.0247	.0155	.0099
16	.8528	.7284	.6232	.5339	.4581	.3936	.3387	.2919	.2519	.2176	.1631	.1229	.1069	.0930	.0708	.0541	.0320	.0193	.0118	.0073
17	.8444	.7142	.6050	.5134	.4363	.3714	.3166	.2703	.2311	.1978	.1456	.1078	.0929	.0802	.0600	.0451	.0258	.0150	.0089	.0054
18	.8360	.7002	.5874	.4936	.4155	.3503	.2959	.2502	.2120	.1799	.1300	.0946	.0808	.0691	.0508	.0376	.0208	.0118	.0068	.0039
19	.8277	.6864	.5703	.4746	.3957	.3305	.2765	.2317	.1945	.1635	.1161	.0829	.0703	.0596	.0431	.0313	.0168	.0092	.0051	.0029
20	.8195	.6730	.5537	.4564	.3769	.3118	.2584	.2145	.1784	.1486	.1037	.0728	.0611	.0514	.0365	.0261	.0135	.0072	.0039	.0021
25	.7798	.6095	.4776	.3751	.2953	.2330	.1842	.1460	.1160	.0923	.0588	.0378	.0304	.0245	.0160	.0105	.0046	.0021	.0010	.0005
30	.7419	.5521	.4120	.3083	.2314	.1741	.1314	.0994	.0754	.0573	.0334	.0196	.0151	.0116	.0070	.0042	.0016	.0006	.0002	.0001
40	.6717	.4529	.3066	.2083	.1420	.0972	.0668	.0460	.0318	.0221	.0107	.0053	.0037	.0026	.0013	.0007	.0002	.0001	*	*
50	.6080	.3715	.2281	.1407	.0872	.0543	.0339	.0213	.0134	.0085	.0035	.0014	.0009	.0006	.0003	.0001	*	*	*	*
60	.5504	.3048	.1697	.0951	.0535	.0303	.0173	.0099	.0057	.0033	.0011	.0004	.0002	.0001	*	*	*	*	*	*

*The factor is zero to four decimal places.

DATA TABLE 3. SUM OF AN ANNUITY OF $1 (FVIF$_a$).

Number of Periods	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%	36%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600	2.0700	2.0800	2.0900	2.1000	2.1200	2.1400	2.1500	2.1600	2.1800	2.2000	2.2400	2.2800	2.3200	2.3600
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3744	3.4396	3.4725	3.5056	3.5724	3.6400	3.7776	3.9184	4.0624	4.2096
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7793	4.9211	4.9934	5.0665	5.2154	5.3680	5.6842	6.0156	6.3624	6.7251
5	5.1010	5.2040	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.3528	6.6101	6.7424	6.8771	7.1542	7.4416	8.0484	8.6999	9.3983	10.146
6	6.1520	6.3081	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	8.1152	8.5355	8.7537	8.9775	9.4420	9.9299	10.980	12.135	13.405	14.798
7	7.2135	7.4343	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	10.089	10.730	11.066	11.413	12.141	12.915	14.615	16.533	18.695	21.126
8	8.2857	8.5830	8.8923	9.2142	9.5491	9.8975	10.259	10.636	11.028	11.435	12.299	13.232	13.726	14.240	15.327	16.499	19.122	22.163	25.678	29.731
9	9.3685	9.7546	10.159	10.582	11.026	11.491	11.978	12.487	13.021	13.579	14.775	16.085	16.785	17.518	19.085	20.798	24.712	29.369	34.895	41.435
10	10.462	10.949	11.463	12.006	12.577	13.180	13.816	14.486	15.192	15.937	17.548	19.337	20.303	21.321	23.521	25.958	31.643	38.592	47.061	57.351
11	11.566	12.168	12.807	13.486	14.206	14.971	15.783	16.645	17.560	18.531	20.654	23.044	24.349	25.732	28.755	32.150	40.237	50.398	63.121	78.998
12	12.682	13.412	14.192	15.025	15.917	16.869	17.888	18.977	20.140	21.384	24.133	27.270	29.001	30.850	34.931	39.580	50.894	65.510	84.320	108.43
13	13.809	14.680	15.617	16.626	17.713	18.882	20.140	21.495	22.953	24.522	28.029	32.088	34.351	36.786	42.218	48.496	64.109	84.852	112.30	148.47
14	14.947	15.973	17.086	18.291	19.598	21.015	22.550	24.214	26.019	27.975	32.392	37.581	40.504	43.672	50.818	59.195	80.496	109.61	149.23	202.92
15	16.096	17.293	18.598	20.023	21.578	23.276	25.129	27.152	29.360	31.772	37.279	43.842	47.580	51.659	60.965	72.035	100.81	141.30	197.99	276.97
16	17.257	18.639	20.156	21.824	23.657	25.672	27.888	30.324	33.003	35.949	42.753	50.980	55.717	60.925	72.939	87.442	126.01	181.86	262.35	377.69
17	18.430	20.012	21.761	23.697	25.840	28.212	30.840	33.750	36.973	40.544	48.883	59.117	65.075	71.673	87.068	105.93	157.25	233.79	347.30	514.66
18	19.614	21.412	23.414	25.645	28.132	30.905	33.999	37.450	41.301	45.599	55.749	68.394	75.836	84.140	103.74	128.11	195.99	300.25	459.44	700.93
19	20.810	22.840	25.116	27.671	30.539	33.760	37.379	41.446	46.018	51.159	63.439	78.969	88.211	98.603	123.41	154.74	244.03	385.32	607.47	954.27
20	22.019	24.297	26.870	29.778	33.066	36.785	40.995	45.762	51.160	57.275	72.052	91.024	102.44	115.37	146.62	186.68	303.60	494.21	802.86	1298.8
21	23.239	25.783	28.676	31.969	35.719	39.992	44.865	50.422	56.764	64.002	81.698	104.76	118.81	134.84	174.02	225.02	377.46	633.59	1060.7	1767.3
22	24.471	27.299	30.536	34.248	38.505	43.392	49.005	55.456	62.873	71.402	92.502	120.43	137.63	157.41	206.34	271.03	469.05	811.99	1401.2	2404.6
23	25.716	28.845	32.452	36.617	41.430	46.995	53.436	60.893	69.531	79.543	104.60	138.29	159.27	183.60	244.48	326.23	582.62	1040.3	1850.6	3271.3
24	26.973	30.421	34.426	39.082	44.502	50.815	58.176	66.764	76.789	88.497	118.15	158.65	184.16	213.97	289.49	392.48	723.46	1332.6	2443.8	4449.9
25	28.243	32.030	36.459	41.645	47.727	54.864	63.249	73.105	84.700	98.347	133.33	181.87	212.79	249.21	342.60	471.98	898.09	1706.8	3226.8	6052.9
26	29.525	33.670	38.553	44.311	51.113	59.156	68.676	79.954	93.323	109.18	150.33	208.33	245.71	290.08	405.27	567.37	1114.6	2185.7	4260.4	8233.0
27	30.820	35.344	40.709	47.084	54.669	63.705	74.483	87.350	102.72	121.09	169.37	238.49	283.56	337.50	479.22	681.85	1383.1	2798.7	5624.7	11197.9
28	32.129	37.051	42.930	49.967	58.402	68.528	80.697	95.338	112.96	134.20	190.69	272.88	327.10	392.50	566.48	819.22	1716.0	3583.3	7425.6	15230.2
29	33.450	38.792	45.218	52.966	62.322	73.639	87.346	103.96	124.13	148.63	214.58	312.09	377.16	456.30	669.44	984.06	2128.9	4587.6	9802.9	20714.1
30	34.784	40.568	47.575	56.084	66.438	79.058	94.460	113.28	136.30	164.49	241.33	356.78	434.74	530.31	790.94	1181.8	2640.9	5873.2	12940.	28172.2
40	48.886	60.402	75.401	95.025	120.79	154.76	199.63	259.05	337.88	442.59	767.09	1342.0	1779.0	2360.7	4163.2	7343.8	22728.	69377.	*	*
50	64.463	84.579	112.79	152.66	209.34	290.33	406.52	573.76	815.08	1163.9	2400.0	4994.5	7217.7	10435.	21813.	45497.	*	*	*	*
60	81.669	114.05	163.05	237.99	353.58	533.12	813.52	1253.2	1944.7	3034.8	7471.6	18535.	29219.	46057.	*	*	*	*	*	*

*FVIFA > 99,999

DATA TABLE 4. PRESENT VALUE OF AN ANNUITY OF $1 ($PVIF_a$).

Number of payments	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	12%	14%	15%	16%	18%	20%	24%	28%	32%
1	0.9901	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9174	0.9091	0.8929	0.8772	0.8696	0.8621	0.8475	0.8333	0.8065	0.7813	0.7576
2	1.9704	1.9416	1.9135	1.8861	1.8594	1.8334	1.8080	1.7833	1.7591	1.7355	1.6901	1.6467	1.6257	1.6052	1.5656	1.5278	1.4568	1.3916	1.3315
3	2.9410	2.8839	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4018	2.3216	2.2832	2.2459	2.1743	2.1065	1.9813	1.8684	1.7663
4	3.9020	3.8077	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.0373	2.9137	2.8550	2.7982	2.6901	2.5887	2.4043	2.2410	2.0957
5	4.8534	4.7135	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6048	3.4331	3.3522	3.2743	3.1272	2.9906	2.7454	2.5320	2.3452
6	5.7955	5.6014	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.1114	3.8887	3.7845	3.6847	3.4976	3.3255	3.0205	2.7594	2.5342
7	6.7282	6.4720	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.5638	4.2883	4.1604	4.0386	3.8115	3.6046	3.2423	2.9370	2.6775
8	7.6517	7.3255	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	4.9676	4.6389	4.4873	4.3436	4.0776	3.8372	3.4212	3.0758	2.7860
9	8.5660	8.1622	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7590	5.3282	4.9464	4.7716	4.6065	4.3030	4.0310	3.5655	3.1842	2.8681
10	9.4713	8.9826	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.6502	5.2161	5.0188	4.8332	4.4941	4.1925	3.6819	3.2689	2.9304
11	10.3676	9.7868	9.2526	8.7605	8.3064	7.8869	7.4987	7.1390	6.8052	6.4951	5.9377	5.4527	5.2337	5.0286	4.6560	4.3271	3.7757	3.3351	2.9776
12	11.2551	10.5753	9.9540	9.3851	8.8633	8.3838	7.9427	7.5361	7.1607	6.8137	6.1944	5.6603	5.4206	5.1971	4.7932	4.4392	3.8514	3.3868	3.0133
13	12.1337	11.3484	10.6350	9.9856	9.3936	8.8527	8.3577	7.9038	7.4869	7.1034	6.4235	5.8424	5.5831	5.3423	4.9095	4.5327	3.9124	3.4272	3.0404
14	13.0037	12.1062	11.2961	10.5631	9.8986	9.2950	8.7455	8.2442	7.7862	7.3667	6.6282	6.0021	5.7245	5.4675	5.0081	4.6106	3.9616	3.4587	3.0609
15	13.8651	12.8493	11.9379	11.1184	10.3797	9.7122	9.1079	8.5595	8.0607	7.6061	6.8109	6.1422	5.8474	5.5755	5.0916	4.6755	4.0013	3.4834	3.0764
16	14.7179	13.5777	12.5611	11.6523	10.8378	10.1059	9.4466	8.8514	8.3126	7.8237	6.9740	6.2651	5.9542	5.6685	5.1624	4.7296	4.0333	3.5026	3.0882
17	15.5623	14.2919	13.1661	12.1657	11.2741	10.4773	9.7632	9.1216	8.5436	8.0216	7.1196	6.3729	6.0472	5.7487	5.2223	4.7746	4.0591	3.5177	3.0971
18	16.3983	14.9920	13.7535	12.6593	11.6896	10.8276	10.0591	9.3719	8.7556	8.2014	7.2497	6.4674	6.1280	5.8178	5.2732	4.8122	4.0799	3.5294	3.1039
19	17.2260	15.6785	14.3238	13.1339	12.0853	11.1581	10.3356	9.6036	8.9501	8.3649	7.3658	6.5504	6.1982	5.8775	5.3162	4.8435	4.0967	3.5386	3.1090
20	18.0456	16.3514	14.8775	13.5903	12.4622	11.4699	10.5940	9.8181	9.1285	8.5136	7.4694	6.6231	6.2593	5.9288	5.3527	4.8696	4.1103	3.5458	3.1129
25	22.0232	19.5235	17.4131	15.6221	14.0939	12.7834	11.6536	10.6748	9.8226	9.0770	7.8431	6.8729	6.4641	6.0971	5.4669	4.9476	4.1474	3.5640	3.1220
30	25.8077	22.3965	19.6004	17.2920	15.3725	13.7648	12.4090	11.2578	10.2737	9.4269	8.0552	7.0027	6.5660	6.1772	5.5168	4.9789	4.1601	3.5693	3.1242
40	32.8347	27.3555	23.1148	19.7928	17.1591	15.0463	13.3317	11.9246	10.7574	9.7791	8.2438	7.1050	6.6418	6.2335	5.5482	4.9966	4.1659	3.5712	3.1250
50	39.1961	31.4236	25.7298	21.4822	18.2559	15.7619	13.8007	12.2335	10.9617	9.9148	8.3045	7.1327	6.6605	6.2463	5.5541	4.9995	4.1666	3.5714	3.1250
60	44.9550	34.7609	27.6756	22.6235	18.9293	16.1614	14.0392	12.3766	11.0480	9.9672	8.3240	7.1401	6.6651	6.2492	5.5553	4.9999	4.1667	3.5714	3.1250

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GLOSSARY

Benefit-cost ratio: Actual net cost savings per year, divided by the net cost savings per year needed to recover the investment.

Capital recovery factor: A number indicating the percentage of an investment's original cost that must be generated each year in net cost savings to recover the original cost.

Internal rate of return: The interest rate that causes NPV to equal zero when used as a discount rate.

Mutually-exclusive: Projects that perform the same function.

Net cost savings: Cost savings per year minus cost per year.

Net present value: The present value of cost savings minus the present value of costs.

Original project: An energy-related system that performs a function for the first time for a given business.

Payback period: The length of time required to obtain net cost savings equal to the initial investment.

Replacement project: A course of action that changes or modifies an energy-related system already in use.